The information content of derivatives for monetary policy

Implied volatilities and probabilities

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Summary

There is much discussion about derivatives at central banks. The main focus is on questions about the impact of the growing use of derivative instruments on the stability of the financial markets and the effectiveness of monetary policy measures. Irrespective of the answers, the information contained in the prices of derivatives can be recovered and used by monetary policy-makers for the monetary policy decision process and operational purposes. Since option prices - unlike, for instance, futures or forward rates - by construction also contain information on the expected price or rate fluctuations of the underlying and in fact also on the probability distributions of future events, the focus of the present study is on options.

First it will be shown that the prices of European options can be computed either directly by assuming a probability distribution for the price of the underlying on maturity or indirectly by means of assuming a random process. Before a new distribution-free method for determining implied probabilities is presented and used, the common indirect procedure is followed: the estimated process parameter, the implied volatility, is derived from LIFFE data for Bund future option prices and subjected to an extensive empirical analysis. The relationship between historical volatilities, various implied volatility measures and - after measuring these - actually realised volatilities (HV, IV, FV) 20, 40, and 60 trading days before the maturity of the options is analysed. The IV measures are calculated from at-the-money calls or puts (CALL, PUT, CALL&PUT), simple or weighted averages (MEAN, KAPPA) of all call options traded on the respective days and from intertemporal averages of at-the-money calls on the last five trading days (FED80, FED60).

As expected, the impact of historical on implied volatilities is significant, but they cannot be considered to be the sole determinant. This is consistent with theoretical considerations that option prices contain market expectations regarding the future price volatility of the underlying. The emperical analysis, however, goes beyond the description of market expectations and examines whether implied volatilities are also suitable for forecasting future volatilities. Forecasts based on implied volatilities about the direction in which the price fluctuations of the underlying would point turn out to be reliable. The results of quantitative forecasts, on the other hand, are disillusioning. Even the highest degree of forecasting reliability obtained for a remaining maturity of 40 trading days is, measured by

the determination coefficient, a mere 44% or less. Moreover, the regression series confirm the superiority of the historical volatilities, which by definition relate to the past, if the forecasting horizon is short. Even though forecasting errors sometimes assume considerable proportions, the t-test for the simple quantitative forecast does not reveal a systematic error for any of the implied volatility measures. This implies that the actual option premiums do not deviate systematically from their fair value; there seem to be no risk premiums.

The findings of the study as to whether players in the option markets might perhaps concentrate on the immediate future are ambivalent: Some implied volatilities measured 60 trading days before maturity of the options explain variations in the volatilities actually realised in the subsequent five trading days (FV5) at a rate of over 60%. On the other hand, the determination coefficient of all other maturities is considerably below the value reached when using FV as a regressant, and this is an indication that market participants tend to be geared to the "long term".

In short, it may be stated that the implied volatilities of Bund future options are useful for describing market expectations regarding the price or rate volatility of the underlying. In addition, they contain in particular information on the subsequent actual trend. For these and other forecasting purposes one of the three IV measures CALL, KAPPA or MEAN should be used which exhibit the best properties.

A possible reason for the low degree of reliability of quantitative forecasts is, above all, the continuous news which necessitate ongoing price or rate adjustments and which considerably complicate forecasts. Another reason is probably that market players, while using the Black-Scholes model as a common language, as it were, for communication purposes, are not convinced of the reliability of the model and therefore manually adjust prices or calculate them using other methods. To avoid these difficulties, a more general option price model is used where there is no need for assuming particular random processes and which is directly based on the probability distribution of the price of the underlying. Thereafter, a new method is presented with the help of which the probabilities implied in a series of option prices can be determined by approximating the first partial derivative of the option price with respect to the strike price. In this way the probabilities expected by market players for the price or rate of the underlying on maturity being within, above or below specific intervals can be calculated. Distribution-free methods used so far for determining implied probabilities, which are based on the method of the Breeden-Litzenberg type - and thus on an approximation of the probability density, are frequently faced with the problem that they can assign to the price or rate interval for which data is available a probability sum of less than 100% only. This makes it necessary to use assumptions regarding how much of the missing probability is to be assigned to which edge of the interval observed. On the other hand, the assumptions are obsolete in the approach presented here as it is based on the approximation of the probability distribution. The calculation of this function immediately shows how much of the probability mass is outside the upper or lower edge.

Technically, advantages of the indicator "implied probabilities" are, besides its forecasting horizon, which depending on the market is up to one year, its daily availability, a fully adequate up-to-dateness for most monetary policy objectives and international comparability. Moreover, an analysis of implied probabilities calculated from options on a future obviates in principle the need for looking at the underlying itself since the expectation value resulting from the implied probabilities is identical with the future price or rate. The future is thus redundant.

Using the risk-neutral probabilities determined, which are revealed preferences, objective statements on market expectations can be made. With their help it is possible not only to determine the future values expected by market players "on average". The approach goes beyond this and makes it possible to calculate quantiles or uncertainty or dispersion measures such as the interquartile range. It is also possible to see whether the implied probabilities are distributed multi-modally, which can prevent misjudgements of market expectations as may occur when using point estimators.

The knowledge derived from implied probabilities can be of major importance for preparing monetary policy measures and determining the best timing of such action. Thus a central bank may intervene in one way or another to reduce "undesirable" uncertainty in a market. Because of the flexibility of the method developed, the type of market, say, money, bond or foreign exchange market, is of secondary importance. Other possible uses are assisting money market management, in particular when deciding whether to use a variable-rate or fixed-rate tender, or as an ex-ante risk measure for banking supervision purposes or risk management. Moreover, implied probabilities also make it possible to check the success of monetary policy measures.

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Welcher Laie wird wohl je verstehen, daß der Verkäufer der Verkaufsoption bei Ausübung der Verkaufsoption durch den Käufer der Verkaufsoption der Käufer der von dem Käufer der Verkaufsoption verkauften Wertpapiere ist?[1]

I. Introduction

There is much discussion about derivatives at central banks. The main focus is on questions concerning the impact of the growing use of derivative instruments on the stability of the financial markets and the effectiveness of monetary policy measures.[2] Irrespective of the answers to these questions, the information contained in the prices of derivatives can be recovered and used by monetary policy makers for the monetary policy decision-making process and operational purposes.

It is true that futures and forward rates which can be calculated on the basis of the respective spot rates and the so-called cost of carry may represent expected values. However, they should not contain more information on market expectations or future spot rates than is included in the individual components of the price formula concerned. The

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1 DEMOLIERE quoted after USCZAPOWSKI (1993), p. 41.

2 See BANK FOR INTERNATIONAL SETTLEMENTS (1994c, 1995), CROCKETT (1995), DEUTSCHE BUNDESBANK (1994).

situation is different in the case of option prices. Although they can also be derived via arbitrage assumptions, they likewise, by virtue of their construction, contain information on the expected price or rate fluctuations of the underlying asset and indeed on the probability distributions of future events. This is why the focus of the present study will be on options.

First, after recalling the relevant theoretical underpinning (chapter II), the following questions will be answered:

- What is the relationship between the implied volatilities calculated from Bund futures option prices and the historical and future volatilities of their underlying assets' prices? (chapter III)

- Can implied volatilities be used reliably not only as an indicator of market expectations but also to forecast future price fluctuations of their underlying asset? (chapter III)

- Are players in the option markets extremely short-term-oriented? (chapter III)

After presenting this paper's centrepiece - a new non-parametric method of determining the probabilities implicit in option prices (chapter IV) which market players ascribe to the price of the respective underlying asset on the expiry date of the option - I shall examine

- which information is obtained if this approach is applied to interest-rate options on German money and bond market instruments (chapter IV) and

- how this complex information can be represented (chapter IV).

The last section (chapter V) summarises the results, and points out possible applications of the indicators previously described.

II. A primer on option pricing theory

An option contract involves different rights and obligations on the part of the two contracting parties; the buyer of an option on an asset acquires, against payment of an option premium (price), the right (but not the obligation) to buy (call option) or sell (put option) a predefined quantity of the asset (underlying asset) at a specified price (strike price) on the expiry date (European-style option) or before the expiry date (American-style option). The option writer, by contrast, is obliged to deliver or accept the asset.[3]

Thus options may be used as hedging instruments, but also for speculative purposes. Hence the buyer of a dollar call would hedge against, or profit from, a rising DM/$ exchange rate. The holder of a call option on a bond would profit from a rising bond price and thus from declining yields.[4] A put on a bond can therefore also be interpreted as insurance against declining security prices or rising yields.

Before information is elicited from option prices in the following chapters, it will be shown, first, how such option premiums can be calculated, and which information is thus included in these variables. For the sake of convenience, let us first consider pay-off A, which the holder of a European-style call option receives when his option expires.[5]

(1) $A = \max(0; F_T - K)$

F_T denotes the price of an arbitrary underlying asset on the expiry day (T) of the option, and K denotes the strike price. If the (current) price F of the underlying asset is below K, the option is "out-of-the-money"; if the values are identical, the option is "at-the-money"; otherwise, it is "in-the-money".[6] If the probabilities with which the underlying contract acquires certain values on the expiry date of the option were known, the expected pay-off could be calculated. The present value of this variable precisely equals price C, which risk-neutral market players would be prepared to pay for a call option.

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[3] See, for example, FASTRICH, HEPP (1991); p. 266.

[4] **Call options** on a bond issue could therefore also be referred to as **interest-rate rate puts**.

[5] In the following, only (de facto) European-style options will be considered.

[6] The premiums of at-the-money and out-of-the-money options consist only of a time value. In addition, in-the-money options have what is known as an intrinsic value of F-K>0 (call options) or K-F>0 (put options). If they are deeply in the money, their premiums consist of their intrinsic values only.

$$
(2) \quad \begin{aligned} C &= e^{-r\tau} E[\max(0, F_T - K)] \\ &= e^{-r\tau} \int_{-\infty}^{+\infty} w(F_T) \max(0, F_T - K) dF_T \end{aligned}
$$

$e^{-r\tau}$ is the relevant discount factor for the residual maturity[7] τ of the option, with r representing the risk-free interest rate and $w(F_T)$ the probability density for the value F_T.[8] E denotes the expected value in a risk-neutral world.[9]

The probabilities missing for the calculation of call or put prices, by means of which the underlying asset of the option acquires a certain value on the expiry date, can be generated, for example, by assuming a random process. This process must meet certain requirements. For instance, the expected growth rate of the spot price of the underlying asset has to have a specific relationship to the risk-free interest rate of corresponding maturity. By contrast, futures prices are expected to exhibit zero growth. If, as in the model developed originally by Black and Scholes (1973) for European-style equity options, a geometric Brownian motion process is assumed, only the annualised standard deviation of the daily logarithmic relative price changes - the volatility σ - unknown *ex ante*, has to be determined to (correctly) price an option. The larger σ is, the more widely the potential values of the underlying asset on the expiry date may be dispersed, with the geometric Brownian motion process giving rise to a logarithmic normal distribution of the random variable "price of the underlying asset on the expiry date of the option". The relationship between the volatility of the random process and the probability density of the random variables thus clearly defined, was used by Black and Scholes to represent C as a function of volatility and other parameters. This option pricing formula has proved to be relatively robust, in spite of some restrictive assumptions - including the log-normal distribution of the prices of the underlying asset (and thus the normal distribution of the returns), a constant, risk-free interest rate r, a constant σ, the non-existence of transaction costs and continuous trade, i.e. there are no price jumps. Moreover, the formula continues to be used by practicioners, with only slight changes being necessary as a rule.

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7 In terms of the number of the residual trading days divided by the number of trading days per year (fixed as 252).

8 F_T denotes, as already mentioned, the price of the underlying asset on the expiry date. T is thus a constant index.

9 The premiums for a European-style put option of the same strike can be derived analogously, or, once C has been calculated, from so-called put-call parity.

In addition, the approach has proved to be so flexible that only slight modifications are needed for it to be also used for European-style options on other underlying assets, such as futures on bond issues. The formula for such interest-rate-related underlying assets, which, by their nature, are more interesting for monetary policy purposes than, say, shares, was derived by Black (1976). By this formula, the fair price of a call option is derived from:[10]

$$(3) \qquad C = e^{-r\tau}\left(F \cdot N\left[\frac{\ln\left(\frac{F}{K}\right)+\frac{\sigma^2}{2}\tau}{\sigma\sqrt{\tau}}\right] - K \cdot N\left[\frac{\ln\left(\frac{F}{K}\right)+\frac{\sigma^2}{2}\tau}{\sigma\sqrt{\tau}} - \sigma\sqrt{\tau}\right]\right),$$

where $N[\cdot]$ represents the corresponding value of the (cumulative) normal distribution, and F the current futures price. Since, but for volatility, all the relevant variables are either predetermined or can be observed in the market at any time, the writer and the buyer of the option only have to agree on the value of the dispersion measure before being able to conclude a trade, because (other things being equal) σ is in an unambiguous relationship to C (or to the price for puts, in the case of put options).

For determining the option price, it is immaterial that the real world is not risk-neutral - after all, many options are purchased precisely for reasons of risk-aversion. Black and Scholes demonstrated the possibility of constructing a riskless and profitless portfolio, without committing resources, consisting of the option, a loan (or an investment) and the underlying asset. In other words: it is possible to duplicate an option synthetically by using the two other components. Since the cost of generating such a synthetic option is known, the option price can be derived. If there is a difference between an option premium and the cost of its artificial complement, market participants will reduce the price difference, notwithstanding their risk preferences, by exploiting the arbitrage possibilities. Hence, as long as options can be duplicated synthetically, option prices can be determined in such a way as if we were living in a risk-neutral world.[11]

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10 See, e.g., GEMMILL (1993), p. 185 f.

11 For a comprehensive account, see e.g. LEHRBASS (1994).

III Implied volatilities

1 Properties of volatility in models of the Black-Scholes type[12]

In the Black-Scholes world, a constant price volatility of the underlying asset is assumed over the entire residual maturity. Actually, however, this variable fluctuates over time. This apparent contradiction may be overcome by regarding the σ used in the price formula as the expected value for the volatility of the underlying asset during the entire residual life of the option. Option writers have to assess volatility correctly in order to be duly recompensed for their risk. On the other hand, buyers will also make efforts to forecast this variable as accurately as possible in order to pay no more than the fair price. Thus, the σ included in the option pricing formula should be a forward-looking variable, in which all the information known and relevant at the particular moment is taken into consideration. If this were not the case, some market players would suffer avoidable losses, a situation which cannot be expected to obtain over the long term.

It is therefore obvious that the **implied** (expected) **volatilities** should be calculated from observable option premiums via an adequate pricing formula, so that forecasts can be made on the **future, actually occurring** price fluctuations[13] of the underlying asset.[14] An advantage of using the information indirectly contained in option prices for forecasting purposes is its immediate availability. It is only necessary to enter a few readily measurable values in order to be able to derive iteratively the implied volatility - hereinafter abbreviated to IV - from an option premium.[15] The influence of new information, which can rapidly be incorporated in the prices of the derivatives anyway owing to the low transaction costs, can thus be represented without any significant time-lag.

Of course, it should not be disregarded that the implied volatilities "only" reflect market expectations, and that the latter, even if they are formed rationally, using all known information, are not bound to be correct in every case. However, in this case the errors should not be systematic. Even so, it is possible that the option market players (mainly

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12 These are understood to mean all models which are derived directly from the Black-Scholes approach.

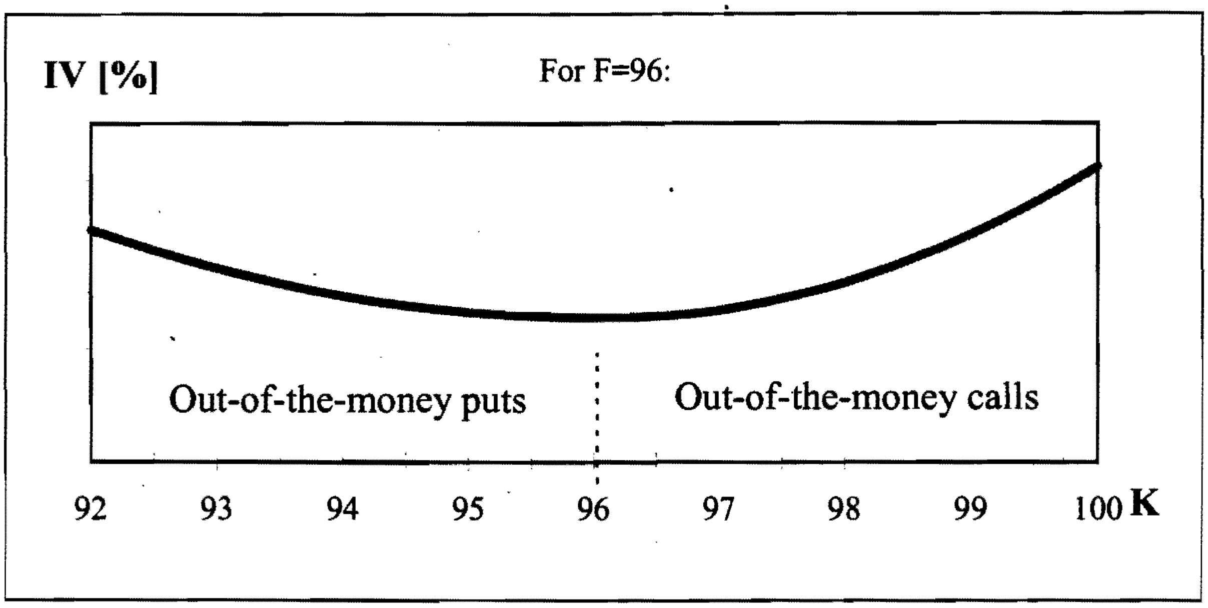
13 Measured like the implied volatility, i.e. as an (annualised) standard deviation of daily price fluctuations.

14 This idea was also the driving force behind the construction of VDAX, introduced on December 5, 1994. This is an index derived from the implied volatilities of DAX options of different expiry dates. For an explanation of the construction of VDAX, see BREUERS (1995).

15 In the case of an option on a bond future, it is sufficient to know the values for C, r, τ, F and K.

professionals) make forecasts of above-average quality, and use them, inter alia, for their volatility estimates. In this case, implied volatilities might help to predict future volatility trends.

Figure 1: Schematic representation of the "volatility smile"



If one calculates implied volatilities for identical options which only exhibit different strike prices, one finds that, for almost all underlyings assets, the IV values depend on K. Generally, a graphical representation (see figure 1) of IV against different K will be found to generate a curve in the form of a skewed smile,[16] which is not consistent with the simple Black-Scholes model. The log-normal distribution of the underlyings' prices assumed there requires identical IV values for all options of one maturity class. Presumably, however, market participants assume that the probability density of the underlying assets' prices is not compatible with a simple log-normal distribution, and that the out-of-the-money options calculated by the Black (-Scholes) procedure would be too cheap, since the probability of major price changes would be understated. Then option-writers would have to demand higher prices, which would be reflected in higher IV values. This price effect may be reinforced, in the case of options running into the money, by the sometimes dramatic increase in gamma, i.e. the second partial derivative of the option price with

[16] The "smile" is normally referred to in the option markets as a "volatility smile".

respect to the price of the underlying asset. Option-writers would have to be reimbursed for this as well.[17]

A possible cause of the lop-sidedness of the smile is, inter alia, a "skewed" probability density for relative price changes (returns). Other conceivable reasons for the smile and its skewness are potential jumps in the price movements of the underlying contract. It may be that the value of the underlying asset jumps from one (normal) distribution to another - for example, owing to political news which is not yet included in the price of the underlying asset. If the directions of the jumps have differing probabilities, puts and calls which are out-of-the-money by an equal distance[18] have different implied volatilities.[19]

It is not possible to say *ex ante* whether and to what extent potential imperfections of the Black-Scholes approach, or exceptional demand situations, affect the information content of implied volatilities.[20] This task can only be performed by an empirical test which, in view of the volatility smile, should take due account of different IV measures.

2 Empirical analysis

In order to clarify the question of whether and to what extent future changes in the volatility of the underlying asset concerned are predictable by using implied volatilities, an analysis is to be conducted below which combines the approaches of studies by the Federal Reserve Bank of Atlanta and the Bank of Japan.[21]

In this context, the futures market for long-term German government bonds will be considered. The volatility of this market deserves much attention, not least because private

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[17] See COOKSON (1993), HULL (1993).

[18] Strictly speaking, the delta of an option indicates the "moneyness" of an option, i.e. how far the option is in-the-money or out-of-the-money.

[19] For these arguments, see COOKSON (1993), MURPHY (1994), MALZ (1994).

[20] For instance, in a discussion Allan Malz drew my attention to the fact that the sky-rocketing of the implied volatilities of DM-US dollar options in spring 1995 was probably attributable not least to the use of knock-out options. This type of option is characterised by the fact that the option irrevocably loses its value if the price of the underlying asset overshoots or undershoots a previously defined limit. Such options are hedged statically with the aid of other options. However, hedging is effective only for a certain price range of the underlying asset. If the options run into-the-money to the extent that this range is left and the options are knocked out, the option-writer still has a "hedge" position which consists of several written in-the-money options. In order to settle these contracts, he has to repurchase this multiple of his knock-out option position in the money. The resulting demand pressure pushes up the option prices excessively, and thus the implied volatilities, too.

[21] See FEINSTEIN (1989) and BANK OF JAPAN (1995).

and government debt tend to be of a long-term nature. In addition, the nominal interest rate may contain, besides the real interest rate and inflation expectations, a volatility premium (as part of the risk premium), by which investors recompense themselves for the higher risk associated with holding volatile assets. In this case, it would be useful if volatility changes were predictable, so as to enable statements to be made on the trends in such a premium.

After examining how reliably an increase or decrease in volatility can be predicted by means of different IV measures, it is to be investigated quantitatively, by means of different error measures, whether implied volatilities are good estimators of future volatilities. Subsequently, a regression analysis will be performed which focuses on the question of the relative advantageousness of the use of historical versus implied volatilities. In addition, a potential short-term orientation of the option market is considered. Thus the question is also addressed of whether the market forms expectations regarding the entire residual maturity of the option or only concerning the next few trading days.

In order to obtain statistically independent forecast errors one should not use the implied volatilities derived from options of the same maturity class which are measured at different points in time.[22] To ensure that there is no overlapping of forecast horizons and, at the same time, that the forecast horizon is as long as possible, a fixed residual maturitiy (tenor) of 60 trading days was initially chosen.[23] To be able to assess the impact of a change in the tenor on the forecasting quality, all tests and regressions were carried out once again with a fixed period of 40 and 20 days before the expiry date, and evaluated separately. Therefore, the results of three test series are available. This is interesting, in particular, since *ex ante* considerations do not permit clear statements to be made on the impact of a declining residual maturity on the quality of forecasts, because there are two different effects. On the one hand, all things being equal, as the expiry day is getting coser, absolute error measures are likely to show smaller errors, simply because there is less to predict. On the other hand, if the residual maturity of an option decreases, its time value is reduced disproportionally, and it increasingly loses its option characteristics. The volatility expected in the future thus may play a diminishing role in price formation.[24]

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22 Example: if - ex post - one calculates the implied volatility of an option which expires on November 23, 1994 both on September 1, 1994 (residual maturity: 60 trading days) and on September 29, 1994 (40 days), the resulting forecasting errors are intercorrelated owing to the temporal overlap.

23 In terms of calender days, this roughly corresponds to one quarter of a year and thus allows for four observation dates per annum - one for each futures contract.

24 These problems will be addressed once again in the next section.

2.1 Data

The empirical analysis is based on data provided by LIFFE. In the context of this study, LIFFE's Bund future contracts have three advantages, in particular: the products quoted there are liquid and have already been traded since the late eighties.[25] Hence, the observation period starts in June 1989 and lasts until November 1994. In addition, options at LIFFE are margined, which has two desirable consequences: firstly, discounting of the option price is dispensed with. In this way the writer is compensated for waiting, since the buyer does not have to pay the option premium in advance.[26] The pricing formula for a call on a future (3) can thus be simplified as follows:

$$(3') \qquad C = F \cdot N\left[\frac{\ln\left(\frac{F}{K}\right) + \frac{\sigma^2}{2}\tau}{\sigma\sqrt{\tau}}\right] - K \cdot N\left[\frac{\ln\left(\frac{F}{K}\right) + \frac{\sigma^2}{2}\tau}{\sigma\sqrt{\tau}} - \sigma\sqrt{\tau}\right]$$

Secondly, owing to the low transaction costs, margining is levelling-out the difference between European-style and American-style options.[27] Hence Black's formula is applicable to LIFFE options, which *de jure* are of the American type.

In order to maintain the simultaneity of all data, which is essential for this study, the closing prices of both Bund futures and options were chosen. Those of the futures were needed for several purposes. They served as input (F) for the calculation of the implied volatilities on the appointed days, in accordance with Black's formula. In addition, the value for the historical volatility (HV) was determined on the basis of the annualised standard deviation of the logarithmic relative closing price changes of the last twenty trading days before the measurement of the IV (see figure 2).

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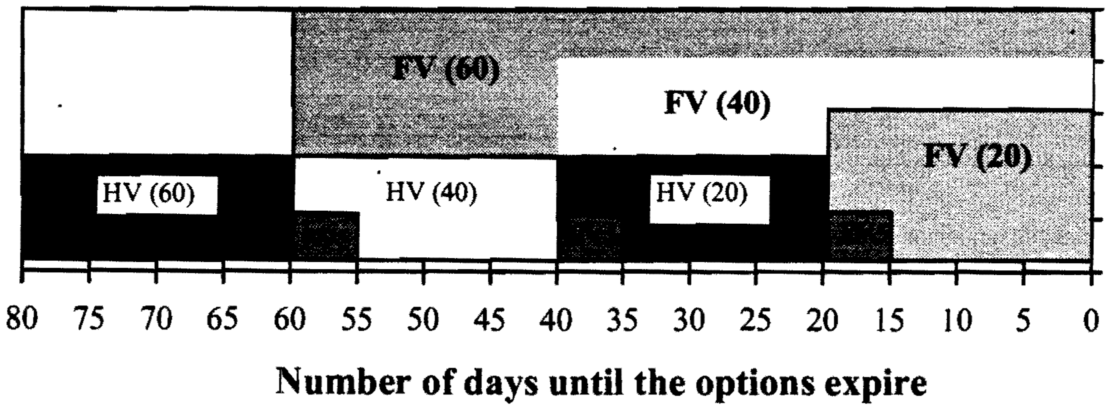
25 In May 1995, LIFFE Bund futures, at a volume of just over 2.6 million contracts, were the most-traded interest rate futures contracts in Europe. See LIFFE (1995).

26 Instead, the buyer has to deposit collateral (for example, bonds), which, however, remain in his possession. Moreover, holding gains are credited to his account daily, while holding losses have to be settled daily as well.

27 Since the build-up and the maintenance of an option position on LIFFE costs next to nothing, a holder will not strike his profit-yielding position owing to the daily margining (marking-to-market), but will keep it and sell it as soon as possible in the event of expected losses. Early exercise is therefore improbable, the difference between European and American-style options thus equal or being near zero. Owing to the negligible difference, LIFFE itself, for example, recommends using Black's formula. See, for instance, LIFFE (1990), particularly p. 19 and GEMMILL (1993), p. 175 ff.

Moreover, the volatility of the underlying futures prices actually realised during the period from the measuring of the IV to the expiry of the option (*ex ante*, this constitutes the future volatility FV), was calculated similarly. However, in this case all closing prices were taken into consideration - i.e. 60, 40 or 20. In addition, to evaluate possible myopic behaviour of players in the option markets, for the tenor of the option being 20, 40 and 60 trading days, respectively the future volatilities were calculated once again; although, only the relative price changes observed during the following five trading days were taken into account. The three time series calculated in this way were designated as FV5.[28]

Figure 2: Calculation periods for HV, FV and FV5



The values of FV are calculated over a period of 60, 40 or 20 trading days. The three FV5 variables include the next 5 relative price changes, which are measured from the 60th, 40th or 20th day before the options expire. The calculation of the historical volatilities is based on 20 closing prices each, measured from the 79th to the 60th, from the 59th to the 40th and from the 39th to the 20th day before the expiry date of the option.

As already mentioned, a whole set of options with different strike prices is being traded at any given time and for any given residual maturity. Theoretical considerations advocate the use of only one option at-the-money,[29] for the deeper it is in-the-money, the more the premium converges towards the intrinsic value of the option. The deeper it is out-of-the

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[28] The calculation of HV, FV and FV5 is geared to the usual approach. See, for example, COX, RUBINSTEIN (1985); p. 255 ff. Thus, all volatility figures are on an annualised basis.

[29] In figure 1, this corresponds to an option with K = 96. Usually, within a given maturity class, at-the-money options have the lowest implied volatilites.

money, the more the premium is "biased" downwards. The information content of the implied variable is lessened in both cases.[30] Since LIFFE, like other exchanges, faces the dilemma of wanting to offer a wide range of potential strike prices and at the same time to bring about a satisfactory liquidity of the individual contracts, only a limited number of strike prices are quoted. Hence, K cannot always be identical with the futures prices. Therefore, it is necessary to fall back on contracts which are just in-the-money or out-of-the-money. For that reason, three series were constructed: the first contains implied volatilities of at-the-money or just out-of-the-money call options (CALL).[31] The second was calculated analogously from puts (PUT),[32] and the third corresponds to the simple average of the first two series (CALL&PUT).

Measuring the implied volatility of a single option has the drawback that a potential error - for example, in ascertaining the closing price of the option - works through in full. For that reason, and in spite of the disadvantages described above, some authors prefer calculating the average from as many contracts as possible. Hence two further series were generated, and only call options with a turnover of more than 40 contracts were included: the first is the simple mean of all the call options traded on the days in question (MEAN).[33] The second IV measure is a weighted average of the same contracts, the weights being identical with the kappa[34] of the options (KAPPA).[35] The idea underlying this procedure is to assign a greater weight to options which respond more strongly to changes in volatility, since market players are likely to price these options more accurately (with respect to σ).

Another suggestion, which seeks to combine the advantages of single at-the-money values with those of average calculation, was put forward by Feinstein of the Federal Reserve

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30 An extreme example of this: a Bund futures option with K=110 and twenty trading days to maturity would have a premium of zero at a futures price of 95, irrespective of whether the expected volatility is two or six per cent per annum.

31 This was proposed by BECKERS (1981).

32 Owing to the put-call parity, the implied volatilities of the out-of-the-money puts are identical to those of in-the-money calls with the same strike price.

33 See SCHMALENSEE, TRIPPI (1978); Common Stock Volatility Expectations Implied by Option Premia, Journal of Finance, Vol. 33, p. 129-147. It can be seen from figure 1 that MEAN always has to turn out larger than, for example, CALL or PUT.

34 The kappa of an option - also called "vega" - is the first (partial) derivative of the premium with respect to the (implied) volatility.

35 See LATANE, RENDLEMAN (1976); Standard Deviations of Stock Price Ratios Implied in Option Prices, Journal of Finance, Vol. 31, p. 369-381. The variable KAPPA is calculated from the same components as MEAN, the volatilities of at-the-money options, however, entering with a greater weight than the volatilities of those contracts which are deeper out-of-the-money or in-the-money.

Bank of Atlanta.[36] For a given day, it proposes to calculate a weighted average of the implied volatilities of at-the-money options of the last five trading days. The most recent option should enter with at least the same weight as its predecessors, since it includes the most recent information. Depending on the weight structure imposed, the influence of the other values gradually decreases with the increasing age of the data, an effect which becomes even more pronounced as the tenor declines as well. Thus, in the case of shorter residual maturities, new information is given greater weight. In accordance with the methods adopted by the Fed Atlanta, two series were calculated, FED60 and FED80. A higher number is accompanied by a higher weighting of the latest at-the-money IV.[37]

2.2 Results of the empirical analysis

The procedures and results of the different analyses are presented in detail, section by section below.

2.2.1 Qualitative forecasts

Qualitative forecasts are intended to help clarify the question of whether implied volatilities can serve at least as a reliable harbinger of the *trend* in future volatility changes. To calculate the trend score, the following procedure was adopted: 60, 40 and 20 days before the expiry of the options, the historical volatility (HV) of the Bund futures and the implied volatilities are compared with each other. **If the implied volatility is higher than the historical, a volatility increase is predicted. If the future volatility** (FV, which is measurable only ex post) of the Bund futures prices until the expiry date of the options **is actually higher** (lower) **than the historical, the** out-of-sample **forecast is classified as correct** (wrong). **An analogus interpretation applies if the implied volatilities are lower than the historical ones**. The individual forecasts can also be reconstructed from figure 3. In that context, the first bar represents the historical volatilities known at the

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36 FEINSTEIN (1989)

37 See formula in the Annex. The CALL measure may be regarded as a special case of the Fed Atlanta weighting, where the most recent implied volatility enters with a weight of 100, the older ones with a weight of zero.

corresponding dates, the same-day implied volatilities can be seen from the data points, and the dark bars represent the volatility FV which actually occurred later on.[38]

The results listed in table 1 - which, like the following ones, are arranged according to the performance of the individual variables - argue in favour of the effectiveness of implied volatilities at forecasting the trend of future volatility changes. This applies particularly to the "medium-term" and "longer-term" maturities, where the best IV measures in more than 90 % and 85 % of the cases, respectively, yield correct forecasts. The difference between the individual indicators with a given tenor is lower than it initially seems: a careful check of the data shows that in the cases in which only individual variables yield incorrect forecasts, the implied volatilities are all distributed very closely around the historical values. Even very slightly changed IV values (as a rule, about ± 0.1 percentage point at an average volatility of just over 5 %) would lead to identical results for the various measures. The small differences between the individual IV measures are also illustrated in figure 3 (above; for τ=60/252). Only for February 1990 and February and June 1994 can the features of CALL and FED80 be differentiated. Since the situation is similar with regard to the other subcharts, only one of the implied variables (i.e. CALL) is shown and represents all others in this context.

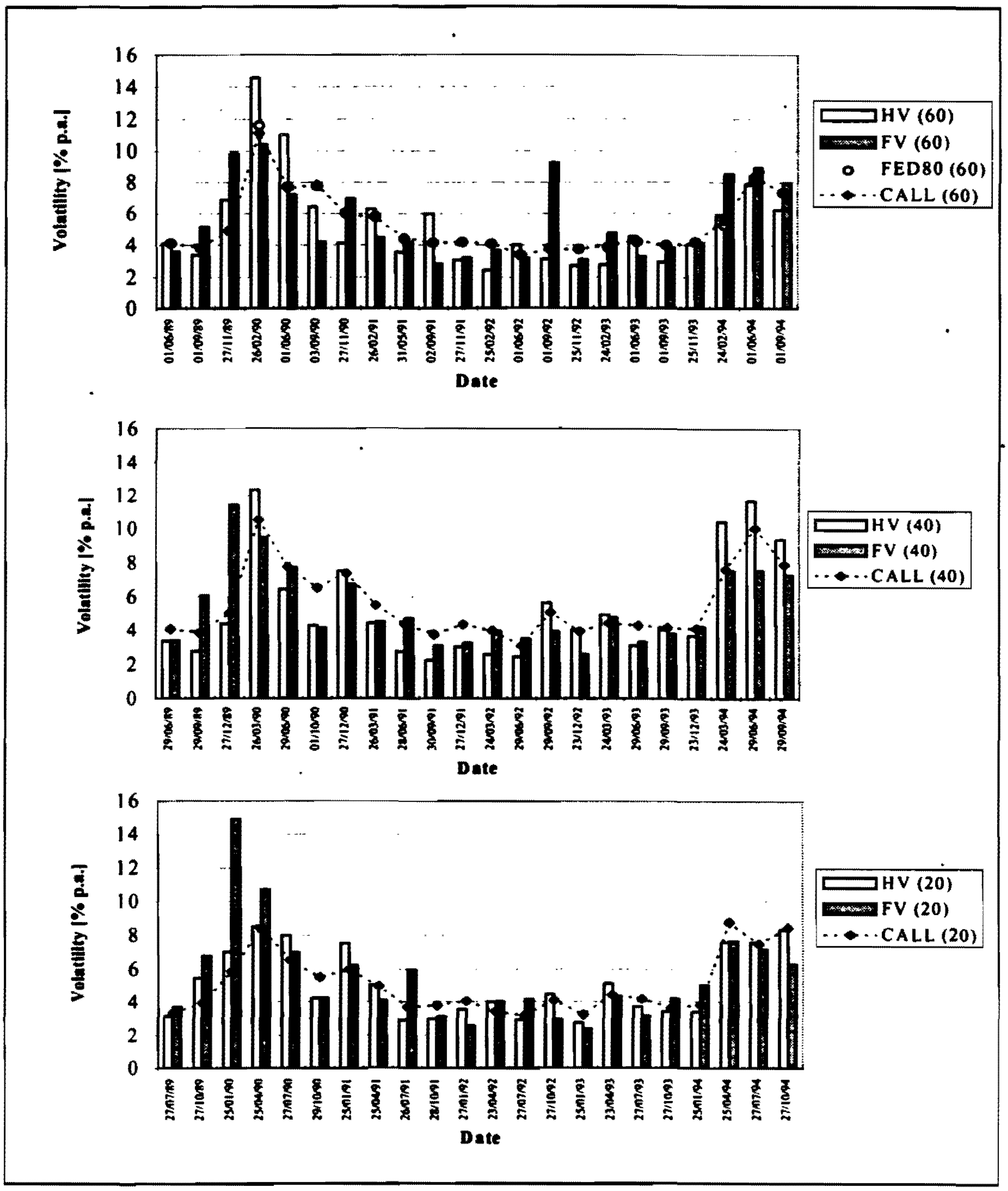
By contrast, in most cases in which none of the IV measures yields a correct signal, the deviations between the individual variables are very considerable. For a tenor of 20 days, this is the case for six out of seven errors. For residual maturities of 40 and 60 days, the differences are perceptible in one and three cases, respectively. In this context, especially the events after the breaching of the Berlin wall, took the financial markets by surprise. The EMS upheavals in the early nineties and the publication of the unexpectedly high monetary growth in January 1994 (on March 2, 1994) were probably also responsible for significant wrong forecasts (as can be seen from figure 3).

[38] Reading example for a residual maturity of 40 trading days on 30/09/1991: the IV measure CALL indicates here, correctly, an increase in the price volatility of the Bund futures during the next 40 days.

Table 1: Qualitative volatility forecasts

60 days			
IV	Number of correct forecasts		
	total	Increase	Decrease
CALL	19	12	7
MEAN	19	12	7
FED60	19	12	7
FED80	19	12	7
KAPPA	19	12	7
CALL&PUT	18	12	6
PUT	18	12	6
of	22	14	8
40 days			
IV	Number of correct forecasts		
	total	Increase	Decrease
CALL	20	12	8
CALL&PUT	20	12	8
FED60	20	12	8
FED80	20	12	8
PUT	20	12	8
MEAN	18	12	6
KAPPA	18	12	6
of	22	12	10
20 days			
IV	Number of correct forecasts		
	total	Increase	Decrease
FED60	15	9	6
FED80	15	9	6
CALL	14	8	6
MEAN	14	9	5
KAPPA	14	8	6
CALL&PUT	13	8	5
PUT	12	8	4
of	22	12	10

Figure 3: **Historical, implied and realised volatilities, with 60, 40 and 20 days to the expiry date**



When considering the charts, a conspicuous feature, apart from the very small difference between the two IV measures, is that the implied volatilities - particularly in periods of low price fluctuations - are perceptibly bigger than the FV predicted by them. This may indicate that option prices contain risk premiums which, in turn, would lead to the implied volatility systematically being calculated too high. However, it still has to be examined whether there is in fact significant "mispricing", which could also be interpreted as a lack of competition or a lack of market efficiency. Owing to the special importance of this problem, also for the chapter on implied probabilities, this issue is to be addressed in more detail.

2.2.2 Simple quantitative forecasts

In order to obtain a clearer picture of the predictive performance of implied volatilities, their deviations from the FV, the volatilities of the Bund futures actually realised later, were calculated. In this context, the "mean squared error" (MSE) and the "mean absolute error" (MAE) served as error measures. In addition, it was checked whether the difference between the estimated and the realised values differs significantly from zero.

When considering the MAE for a tenor of sixty days (table 2), it is striking, first of all, that the forecasting quality of the IV measures is approximately uniform. All the values are between 1.33 and 1.39 percentage points. However, this is a remarkable magnitude. Given a mean value of just under 5.6 % for FV, this is accompanied by a percentage error of as much as 24 %. Even more serious, however, is the lack of precision of the forecasts on the basis of historical volatilities. The mean absolute error of 1.9 percentage points is equivalent to 34 % of the average volatility.

If the forecast horizon is reduced from 60 to 40 trading days, both the mean absolute error and the squared error decline. A further reduction to 20 trading days, by contrast, worsens the precision of the forecast. In relative terms, the picture now changes in favour of the historical variable, which exhibits lower predictive errors as the tenor declines. One of the two reasons for this might be the decreasing size of the sample for generating the FV. Shocks or random processes which shaped the pattern of the historical volatilities and the impact of which continues even after the arbitrary demarcation between HV and FV are now perhaps reflected more clearly in the latter variable than is the case when the acutally realised volatilities are calculated using more (i. e. 40 or 60) futures prices. In addition, the time value of options decreases disproportionately fast as the time to expiry declines. The options increasingly lose their option character, and their premiums more and more converge towards their intrinsic value. In other words: the expected volatility plays an ever-decreasing role in price formation.

Table 2: Accuracy of quantitative forecasts

MAE					
60 days		40 days		20 days	
PUT	1.33	CALL	1.10	HV	1.31
CALL&PUT	1.33	KAPPA	1.12	MEAN	1.41
CALL	1.33	MEAN	1.13	FED60	1.42
KAPPA	1.36	CALL&PUT	1.13	FED80	1.42
MEAN	1.37	FED80	1.14	KAPPA	1.43
FED80	1.38	PUT	1.15	CALL	1.45
FED60	1.39	FED60	1.16	CALL&PUT	1.46
HV	1.93	HV	1.57	PUT	1.46
MSE					
60 days		40 days		20 days	
MEAN	3.97	CALL	3.07	HV	4.33
KAPPA	3.99	KAPPA	3.09	MEAN	4.99
CALL	4.00	MEAN	3.11	KAPPA	5.15
CALL&PUT	4.04	CALL&PUT	3.22	FED60	5.24
PUT	4.08	FED80	3.24	FED80	5.27
FED80	4.10	FED60	3.28	CALL	5.36
FED60	4.14	PUT	3.38	CALL&PUT	5.41
HV	5.86	HV	5.20	PUT	5.46
Bias					
60 days		40 days		20 days	
HV	-0.49	HV	-0.10	MEAN	-0.72
MEAN	-0.61	CALL	0.45	KAPPA	-0.89
PUT	-0.63	CALL&PUT	0.46	HV	-0.92
KAPPA	-0.65	FED60	0.47	PUT	-1.05
CALL&PUT	-0.65	FED80	0.47	FED60	-1.07
FED60	-0.66	PUT	0.47	FED80	-1.10
FED80	-0.67	KAPPA	0.57	CALL&PUT	-1.10
CALL	-0.67	MEAN	0.66	CALL	-1.15

MAE: Mean absolute error. MSE: Mean squared error. Bias: t statistic

The examination of the forecast errors for a bias yielded two interesting results. First, no statistically significant distortion is discernible. Second, the minus sign in the case of a residual maturity of 60 and 20 days, respectively, suggests that the implied volatilities, taking the average of the years between 1989 and 1994, tended to be too low. This finding therefore contradicts the previously formulated hypothesis that there is a risk premium. The large number of observations indicating that the implied volatilities, and thus the prices, are slightly too high are apparently accompanied by a few, albeit very distinct, errors in the other direction. Hence there can be no talk of systematic "mispricing".[39]

If the relative performances of several variables are compared, the bad performance of the intertemporal weighted averages FED60 and FED80 is striking. The additional programming and computing effort for generating these variables does not seem justified, since they usually appear in the lower half of the "ranking lists". This also applies to the implied volatilities calculated from at-the-money put options.

2.2.3 Regression analysis

There are two things you are better off not watching in the making: sausages and econometric estimates.[40]

The previous "test" demonstrates the low reliability of "naive" quantitative forecasts. In order to shed even more light on the predictive power and the superiority of implied volatilities versus historical volatilities, a regression analysis is now carried out for each of the three residual maturities.

2.2.3.1 The explanatory power of implied and historical volatilities

Initially the following equation is to be estimated; in this context, the OLS method can be employed since, by design, there is no temporal overlapping of the variables:

(4) $ZV_t = a + b \cdot Volatility_t + \varepsilon_t$

[39] This applies all the more since only individual points of time are considered here. For statements about risk premiums, the observation of the premiums of individual options over time would also be advisable.

[40] Origin unknown. Source: LEAMER (1983).

"Volatility" denotes one of the IV measures or HV, as the case may be. The variable "ε" represents a normally distributed homoscedastic disturbance variable; a and b are parameters to be estimated.

If all the information available at time t is processed rationally and if implied volatilities are used as the regressor, the estimated value of the constant, should not be significantly different from zero. Moreover, the coefficient b should not differ significantly from one. In addition, the disturbance term should be free from serial correlation, for systematic errors are inconsistent with rational expectations. If the estimated value of b is lower than one but significantly larger than zero, the corresponding exogenous variable nevertheless has an explanatory power with regard to FV_t.

If HV is used as an explanatory variable, even systematic errors are not ruled out, as by definition HV is backward-looking only.

Table 3: Hypotheses to be tested for rational information processing

Hypothesis	reject
a=0	no
b=0	yes
b=1	no

The DW statistics listed in table 4 do not provide any indication of first-order serial correlation in the case of 60 days to the maturity of the option. With 40 days to expiry, the picture remains the same. Only if HV is used as an independent variable is the DW statistic in the undetermined area. Also with regard to the significance of the coefficients, the long-term and medium-term forecast horizons (i.e. $\tau = 60/252$ and $\tau = 40/252$) differ only slightly. Statistically speaking, the constant does not deviate significantly from zero for any IV measure. The other two significance tests likewise do not argue against rational expectations: The estimated coefficients, none of which is greater than 0.8, all differ to a statistically significant extent from zero, but not from one. As expected, only HV fails to satisfy this criterion.

Like the results outlined in the previous sections, the adjusted R^2 implies greater predictive accuracy over the medium term than over the longer forecast horizon. Whereas in the 60 days case the implied volatilities, which were calculated on the basis of at-the-money (or just out-of-the-money) call options, can explain barely 36 % of the variation of the future volatilities, the share is more than 8 percentage points higher in the case of 40 days. However, this simple estimation approach apparently does not generate satisfactory results for the reliable quantification of future price fluctuations by means of implied volatilities alone.

Table 4: Explanatory power of implied and historical volatilities for FV

60 days

Volatility	a	b	R^2	DW	t value (b=1)
CALL	1.405 (1.251)	0.789 ** (0.222)	0.357	1.990	-0.954
MEAN	1.341 (1.269)	0.796 ** (0.224)	0.356	1.963	-0.909
KAPPA	1.356 (1.270)	0.796 ** (0.225)	0.354	1.964	-0.909
CALL&PUT	1.449 (1.249)	0.779 ** (0.221)	0.353	1.991	-0.998
FED80	1.551 (1.228)	0.762 ** (0.217)	0.350	1.978	-1.097
PUT	1.494 (1.247)	0.770 ** (0.220)	0.348	1.992	-1.045
FED60	1.592 (1.226)	0.754 ** (0.217)	0.346	1.962	-1.138
HV	2.831 ** (0.918)	0.521 ** (0.153)	0.336	1.999	-3.131 **

20 days

Volatility	a	b	R^2	DW	t value (b=1)
HV	0.448 (1.244)	0.991 ** (0.228)	0.460	1.465	-0.041
MEAN	0.105 (1.569)	1.032 ** (0.286)	0.364	1.215 *	0.112
KAPPA	0.384 (1.539)	0.992 ** (0.283)	0.349	1.228 *	-0.029
FED60	0.660 (1.472)	0.954 ** (0.274)	0.346	1.322 U	-0.166
FED80	0.651 (1.482)	0.957 ** (0.276)	0.344	1.301 U	-0.155
CALL	0.650 (1.510)	0.960 ** (0.283)	0.334	1.243 U	-0.140
CALL&PUT	0.730 (1.509)	0.942 ** (0.282)	0.326	1.231 *	-0.207
PUT	0.815 (1.508)	0.922 ** (0.280)	0.319	1.220 *	-0.279

40 days

Volatility	a	b	R^2	DW	t value (b=1)
CALL	1.159 (1.065)	0.754 ** (0.180)	0.440	1.535	-1.365
KAPPA	1.083 (1.086)	0.760 ** (0.183)	0.438	1.512	-1.313
MEAN	1.033 (1.101)	0.764 ** (0.184)	0.436	1.474	-1.281
FED80	1.317 (1.060)	0.724 ** (0.179)	0.423	1.552	-1.548
CALL&PUT	1.278 (1.073)	0.731 ** (0.181)	0.421	1.518	-1.482
FED60	1.354 (1.057)	0.717 ** (0.178)	0.420	1.563	-1.593
HV	2.699 ** (0.770)	0.500 ** (0.127)	0.408	1.309 U	-3.937 **
PUT	1.398 (1.080)	0.709 ** (0.182)	0.403	1.501	-1.599

20 days - corrected estimations

Volatility	a	b	R^2	DW	t value (b=1)
HV	1.272 (1.782)	0.823 ** (0.315)	0.462	1.810	-0.041
MEAN	0.946 (2.362)	0.860 ** (0.410)	0.417	1.804	0.112
KAPPA	1.275 (2.338)	0.811 ** (0.409)	0.402	1.792	-0.029
CALL	1.649 (2.297)	0.758 ** (0.408)	0.386	1.782	-0.140
CALL&PUT	1.705 (2.292)	0.744 ** (0.405)	0.382	1.785	-0.207
FED80	1.694 (2.339)	0.747 ** (0.418)	0.382	1.803	-0.155
FED60	1.718 (2.355)	0.742 ** (0.422)	0.380	1.811	-0.166
PUT	1.765 (2.291)	0.730 ** (0.402)	0.378	1.787	-0.279

R^2: adjusted coefficient of determination; DW: Durbin-Watson statistic; *,**: significant at the level of 5 or 1% (error level); U: DW is in the undetermined area; figures in brackets: standard errors of the estimates

If the tenor of the option is reduced once again, the t values have to be interpreted with caution in almost all equations, since there are indications of first-order serial correlation. In the estimation adjusted for this (table 4, bottom right-hand corner), the implied volatilities come off badly, not only in absolute terms but also, measured by R^2, relative to their historical counterparts, even though all the estimated parameters meet the pertinent requirements with regard to rational information processing. Even if the most reliable IV measure is used, the adjusted coefficient of determination is only 0.42; the vast bulk is significantly lower. By contrast, HV, at just over 0.46, comes off perceptibly better.

In a further series of investigations it is to be tested whether the participants in option markets are "short sighted". The analysis is intended to show which of the expected price fluctuations play a greater part on price formation, those for the next few days or those for the entire residual maturity of the option. For this purpose, the regressions described above are carried out once again, but this time using FV5 as the endogenous variable. The regressors are once again the implied or historical volatilities measured 60, 40 or 20 days before the expiry of the options.

Only in one of the 24 regressions run (see table 5) does the DW statistic show a potential problem with serial correlation in the error term. A re-estimation of the corresponding equation, using an appropriate adjustment procedure, resulted in only minor changes, and is therefore not included in the table.

If IV and HV are measured 60 days before the expiry of the options, all the estimated values behave just as is required by the ideal case described at the beginning of this section - b differs significantly from zero, but not from one, and a does not differ from zero. In addition, the variation in future volatility measured only over the next few days (i. e. FV5), at partly more than 63 %, can be predicted considerably better than the future volatility measured over the whole remaining life of the option (i. e. FV). That suggests a short-term orientation of the option market.

Strangely enough, this property cannot be found in the other two regression series. Instead, the adjusted R^2 was almost 30 percentage points lower in the 40 days case, and thus for FV5 was about 10 percentage points lower than for the explanation of FV. In addition, all the estimations for b differ significantly from one (error probability $\leq$ 5 %).

Table 5: Explanatory power of implied and historical volatilities for FV5

Volatility	a	b	R^2	DW	t value (b=1)
60 days					
MEAN	-2.474	1.428 **	0.636	1.406 U	1.838
	(1.318)	(0.233)			
CALL	-2.332	1.408 **	0.631	1.561	1.762
	(1.308)	(0.232)			
KAPPA	-2.432	1.424 **	0.629	1.445	1.802
	(1.328)	(0.235)			
CALL&PUT	-2.256	1.392 **	0.625	1.582	1.690
	(1.313)	(0.232)			
PUT	-2.177	1.376 **	0.618	1.602	1.617
	(1.317)	(0.233)			
FED80	-1.978	1.343 **	0.603	1.495	1.465
	(1.324)	(0.234)			
FED60	-1.880	1.324 **	0.591	1.476	1.370
	(1.338)	(0.236)			
HV	0.549	0.868 **	0.512	1.442	-0.729
	(1.086)	(0.181)			
40 days					
MEAN	1.270	0.605 **	0.346	2.166	-2.275 *
	(1.039)	(0.174)			
KAPPA	1.328	0.599 **	0.344	2.171	-2.322 *
	(1.028)	(0.173)			
FED60	1.499	0.572 **	0.340	2.180	-2.574 *
	(0.988)	(0.166)			
FED80	1.485	0.574 **	0.338	2.176	-2.539 *
	(0.995)	(0.168)			
CALL	1.424	0.587 **	0.336	2.170	-2.402 *
	(1.015)	(0.172)			
CALL&PUT	1.529	0.567 **	0.319	2.152	-2.512 *
	(1.020)	(0.172)			
PUT	1.634	0.548 **	0.302	2.135	-2.622 *
	(1.024)	(0.172)			
HV	2.776 **	0.360 **	0.260	1.983	-5.139 **
	(0.755)	(0.125)			
20 days					
PUT	1.627	0.528 **	0.304	2.245	-2.851 **
	(0.891)	(0.166)			
CALL&PUT	1.646	0.526 **	0.294	2.218	-2.812 *
	(0.903)	(0.169)			
CALL	1.670	0.523 **	0.283	2.190	-2.781 *
	(0.915)	(0.172)			
KAPPA	1.594	0.527 **	0.279	2.165	-2.717 *
	(0.947)	(0.174)			
MEAN	1.519	0.534 **	0.275	2.156	-2.612 *
	(0.979)	(0.178)			
FED80	1.847	0.486 *	0.247	2.165	-2.970 **
	(0.927)	(0.173)			
FED60	1.907	0.474 *	0.236	2.156	-3.038 **
	(0.930)	(0.173)			
HV	2.209 *	0.412 *	0.208	1.916	-3.647 **
	(0.880)	(0.161)			

R^2: adjusted coefficient of determination; DW: Durbin-Watson statistic
*,**: significant at a level of 5 or 1% (error level); U: DW is in the undetermined area; figures in brackets: standard errors of the estimates.

If τ is reduced to 20/252, the result, just as for the other maturities, is that HV has the lowest explanatory power which is somewhat surprising in the light of the previous considerations and results of the analysis. Hence the hypothesis that the shocks and random processes which shape historical volatility also affect future volatility, and that HV should therefore exhibit a good predictive performance in the case of shorter forecast horizons, cannot be confirmed, at least in this context. All in all, the empirical evidence with regard to a short-term orientation of the option market, given the determination coefficients in the individual tests, presents a mixed picture. In relative terms, the implied variables in the case of the very short-term forecasts of FV5 perform better than in the case of the corresponding projections of FV. However, the R^2 are low, in absolute terms, and the statistical properties of the estimated parameters do not suggest that market players form rational expectations.

2.2.3.2 The influence of historical volatilities on implied volatilities

The next regression approach, namely

(5) $IV_t = a + b \cdot HV_t + \varepsilon_t$

is intended to help evaluating the impact of historical volatilities on implied volatilities. If H_0:b=1 is not rejected, IV can perhaps be explained by HV alone, and is thus not more useful than HV. If H_0:b=1 and H_0:b=0 are rejected, the result suggests that IV contains both information from HV and information from other origins.

Table 6 shows that the change in implied volatilities for all three τ can be explained in large part by the historical variables. In the case of a tenor of 60 days, the explanatory power is more than three-quarters (as measured by R^2). For the medium-term tenor it actually exceeds 90 % (if the estimates adjusted for serial correlation are used). Other than that, the results over the medium and the long term are similar once again. The constant a differs from zero in each case. The same applies to the coefficient b, which is approximately 0.6 in each case, and is always significantly smaller than one.

If the focus is on implied volatilities measured 20 days before the options expire, the picture changes. Only the IV measures CALL, CALL&PUT, KAPPA and MEAN lead to estimated values of b which differ significantly from unity (5 % level). For the other IV measures, it is not very unlikely that their changes can be explained by the variation of the historical volatilities alone.

Table 6: Impact of historical volatilities on implied volatilities

60 days

Volatility	a	b	R^2	DW	t value (b=1)
FED60	1.987 ** (0.420)	0.627 ** (0.070)	0.791	1.909	-5.337 **
MEAN	2.156 ** (0.403)	0.601 ** (0.067)	0.790	1.889	-5.946 **
FED80	2.005 ** (0.419)	0.623 ** (0.070)	0.789	1.904	-5.408 **
KAPPA	2.155 ** (0.407)	0.598 ** (0.068)	0.786	1.828	-5.933 **
CALL	2.127 ** (0.423)	0.601 ** (0.070)	0.774	1.716	-5.667 **
CALL&PUT	2.122 ** (0.431)	0.603 ** (0.072)	0.768	1.671	-5.535 **
PUT	2.117 ** (0.439)	0.605 ** (0.073)	0.763	1.628	-5.401 **

20 days

Volatility	a	b	R^2	DW	t value (b=1)
MEAN	1.144 ** (0.400)	0.800 ** (0.073)	0.849	2.111	-2.722 *
KAPPA	1.004 * (0.419)	0.814 ** (0.077)	0.842	2.126	-2.426 *
CALL&PUT	0.853 (0.449)	0.825 ** (0.082)	0.826	1.965	-2.123 *
CALL	0.879 (0.445)	0.817 ** (0.082)	0.826	2.076	-2.245 *
PUT	0.827 (0.456)	0.833 ** (0.084)	0.824	1.853	-1.993
FED80	0.837 (0.457)	0.828 ** (0.084)	0.822	2.295	-2.051
FED60	0.822 (0.463)	0.832 ** (0.085)	0.820	2.357	-1.976

40 days

Volatility	a	b	R^2	DW	t value (b=1)
CALL&PUT	2.100 ** (0.313)	0.654 ** (0.052)	0.884	0.925 **	-6.719 **
PUT	2.068 ** (0.317)	0.661 ** (0.052)	0.883	0.987 **	-6.491 **
CALL	2.132 ** (0.310)	0.646 ** (0.051)	0.883	0.866 **	-6.922 **
FED80	2.071 ** (0.324)	0.660 ** (0.053)	0.878	0.993 **	-6.359 **
FED60	2.054 ** (0.329)	0.664 ** (0.054)	0.877	1.041 *	-6.202 **
KAPPA	2.235 ** (0.317)	0.637 ** (0.052)	0.875	0.786 **	-6.946 **
MEAN	2.312 ** (0.325)	0.629 ** (0.054)	0.867	0.684 **	-6.907 **

40 days - corrected estimations

Volatility	a	b	R^2	DW	t value (b=1)
MEAN	2.543 ** (0.480)	0.594 ** (0.049)	0.920	2.168	-8.331 **
CALL	2.446 ** (0.451)	0.594 ** (0.050)	0.919	2.097	-8.154 **
KAPPA	2.508 ** (0.458)	0.593 ** (0.050)	0.918	2.147	-8.185 **
CALL&PUT	2.410 ** (0.448)	0.602 ** (0.052)	0.915	2.107	-7.678 **
PUT	2.367 ** (0.443)	0.611 ** (0.054)	0.910	2.124	-7.183 **
FED80	2.327 ** (0.450)	0.617 ** (0.056)	0.905	2.178	-6.865 **
FED60	2.304 ** (0.451)	0.621 ** (0.058)	0.900	2.190	-6.580 **

R^2: adjusted coefficient of determination; DW: Durbin-Watson statistic; *,**: significant at the level of 5 or 1% (error level);
U: DW is in the undetermined area; figures in brackets: standard errors of the estimates

2.2.3.3 Encompassing test

The previous section showed the importance of historical volatilities for explaining the variation of their implied counterparts. However, with a few exceptions in the case of 20 days, implied volatilities cannot be explained by historical volatilities alone. This raises the question of whether future volatilities should rather be estimated through the simultanous use of both the variables known at time t. For this purpose, what is known as an encompassing test is carried out, in which, first, the following equation is estimated:

(6) $ZV_t = a + b \cdot IV_t + c \cdot HV_t + \varepsilon_t$

Subsequently, the significance of the estimated values of b and c is tested. The results are classified in four categories (see table 7),

Table 7: Categories for the encompassing test[41]

	H_0: c = 0 not rejected	H_0: c = 0 rejected
H_0: b = 0 not rejected	Category I	Category II
H_0: b = 0 rejected	Category III	Category IV

which, in the case of a high coefficient of determination or a high F statistic, can be interpreted as follows:

Category I: Each of the measures contains all relevant information. Thus, owing to collinearity, one of the two variables is superfluous.

Category II: All relevant information is contained in the historical volatilities. The inclusion of the implied volatilities does not improve the forecast.

Category III: All the relevant information is contained in the implied volatilities. The inclusion of the historical volatilities does not improve the forecast.

Category IV: Both regressors contain important information which is not contained in the other variable concerned. Thus, both should be used jointly for the forecast.

If R^2 is small, the statements made above have to be modified somewhat. In category I it could also mean that neither of the variables is useful. If only one coefficient differs from zero, this implies the necessity of finding other variables in order to be able to predict the future volatilities with sufficient reliability, since not all relevant information is contained in the corresponding regressor. In the fourth case, too, further variables would have to be found. However, in the categories II-IV, at least, it would have been shown that the significant variables have an information content which is useful for forecasting purposes.

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41 See BANK OF JAPAN (1995).

Table 8: Encompassing Test I (dependent variable: FV)

Volatility	a	b	c	R^2	DW	t value (b=1)	t value (c=1)
60 days							
CALL	1.767	0.501	0.221	0.339	2.035	-1.031	-2.370 *
	(1.379)	(0.485)	(0.329)				
CALL&PUT	1.817	0.478	0.233	0.337	2.039	-1.096	-2.356 *
	(1.365)	(0.476)	(0.325)				
MEAN	1.733	0.509	0.216	0.336	2.008	-0.964	-2.296 *
	(1.430)	(0.509)	(0.342)				
KAPPA	1.756	0.499	0.223	0.336	2.012	-0.993	-2.296 *
	(1.423)	(0.505)	(0.338)				
PUT	1.867	0.455	0.246	0.335	2.043	-1.164	-2.343 *
	(1.351)	(0.468)	(0.322)				
FED80	1.905	0.462	0.234	0.332	2.019	-1.095	-2.239 *
	(1.348)	(0.491)	(0.342)				
FED60	1.958	0.439	0.246	0.330	2.010	-1.141	-2.191 *
	(1.343)	(0.491)	(0.344)				
40 days							
CALL	1.427	0.597	0.114	0.413	1.507 U	-0.729	-2.334 *
	(1.408)	(0.554)	(0.380)				
KAPPA	1.413	0.575	0.134	0.412	1.488 U	-0.784	-2.359 *
	(1.433)	(0.541)	(0.367)				
MEAN	1.415	0.555	0.151	0.411	1.461 U	-0.843	-2.389 *
	(1.442)	(0.528)	(0.355)				
FED80	1.731	0.467	0.191	0.401	1.492 U	-0.996	-2.153 *
	(1.352)	(0.535)	(0.376)				
FED60	1.786	0.445	0.205	0.400	1.493 U	-1.052	-2.131 *
	(1.333)	(0.528)	(0.373)				
CALL&PUT	1.729	0.462	0.198	0.399	1.467 U	-0.969	-2.084
	(1.401)	(0.555)	(0.385)				
PUT	2.018	0.329	0.283	0.388	1.424 U	-1.214	-1.852
	(1.386)	(0.553)	(0.387)				
20 days							
PUT	0.784	-0.406	1.329 *	0.444	1.592	-2.273 *	0.583
	(1.362)	(0.619)	(0.565)				
CALL&PUT	0.755	-0.359	1.287 *	0.441	1.576	-2.159 *	0.505
	(1.374)	(0.630)	(0.569)				
CALL	0.717	-0.306	1.241 *	0.439	1.558	-2.050	0.422
	(1.386)	(0.637)	(0.570)				
KAPPA	0.727	-0.277	1.216	0.437	1.553	-1.883	0.361
	(1.442)	(0.678)	(0.599)				
FED80	0.626	-0.213	1.167	0.435	1.522 U	-1.947	0.295
	(1.375)	(0.623)	(0.566)				
FED60	0.598	-0.182	1.142	0.434	1.511 U	-1.921	0.253
	(1.370)	(0.615)	(0.563)				
MEAN	0.669	-0.193	1.145	0.434	1.525 U	-1.678	0.236
	(1.511)	(0.711)	(0.615)				

R^2: adjusted coefficient of determination; DW: Durbin-Watson statistic; *,**: significant at the level of 5 or 1% (error level); U: DW is in the undetermined area; figures in brackets: standard errors of the estimates.

In addition, especially in cases where b or c are not significantly different from zero, attention should be paid to whether they differ significantly from one. If not, it is still possible that the respective variable has explanatory power for FV.

If future volatility over the whole residual maturity of an option is regressed on implied volatility and on historical volatility calculated from the last twenty closing prices, the results listed in table 8 are obtained.

If FV is calculated from 60 closing prices, it is striking that the value of the adjusted R^2 is always smaller than in the case of a regression with only one of the two exogenous variables (see table 4). The additional explanatory power provided by the second regressor is therefore not great enough to offset the loss of degrees of freedom. In addition, none of the estimated coefficients now deviates significantly from zero. This could be attributable to the collinearity of the regressors. However, owing to their high standard error, the estimated values of b, unlike those of c, do not differ significantly from one, either. Although the results should therefore be classified formally in category I, it can be concluded, bearing in mind the previous results, that

(i) implied volatilities are superior to historical ones,
(ii) the additional consideration of the latter is not useful, and
(iii) although implied volatilities do have some explanatory power, it is not sufficient for a reliable quantitative forecast in the 60 days case. This is true even for the relatively successful measures, such as CALL, MEAN or KAPPA.

These statements are also borne out when there are only 40 days left until the option expires. The results are quite similar to the case described above. Only for the measures CALL&PUT and PUT is it possible to discern a perceptible difference. For them, H_0:c=1 is not rejected. The t values are not affected by serial correlation. Although for this maturity all DW statistics are in the "fuzzy" area, neither correlograms nor the Breusch-Godfrey Lagrange multiplier test (LM test), which was carried out as well, point to serial correlation of any order. Hence, no adjustment was made.

If the tenor of the option is reduced to 20 days, not every regression equation formally belongs to the first category. As the implied volatilities are inferior to the historical ones in this maturity class and do not possess any additional explanatory power the equations in which PUT, CALL&PUT and CALL are the regressors then belong to the second category.

Moreover, in the case of PUT and CALL&PUT the hypothesis that these IV variables incorporate all the necessary information (H_0:b=1) is directly rejected.

If the regressions are carried out once again with FV5 as dependent variable (table 9), the picture shifts to the disadvantage of HV in the case of data collection about one calendar month before the expiry of the Bund futures options (table 9, bottom). Now none of the estimated parameters deviates significantly from zero. The coefficient c, indeed, is always significantly smaller than one. Moreover, the explanatory power is lower, all in all, than in the case of the regression of FV on the same set of variables.

This is also true for t = 40/252. The respective equations likewise belong to category I.

If FV5 is regressed on the implied and historical volatilities measured about three months before the expiry of the options, the IV are apparently markedly superior to the historical volatilities: With the exception of the intertemporal average FED60, all the regressions are to be assigned to the third category. In addition, in the case of these forecasts, just as in the case of the simple regressions of FV5 on IV, the adjusted R^2 exhibits high values. The possible objection concerning the validity of the test statistics in the light of the values of the Durbin-Watson test can once again be countered by arguing that neither the correlogram nor the LM test give any indication of serial correlation of any order.[42]

To sum up, the encompassing tests for future volatilities (FV and FV5), 60 or 40 days before expiry of the options, have thus provided evidence of the superiority of implied volatilities over their historical counterparts. However, the informative value of this finding is advertly affected by the fact that, in formal terms, the results have to be assigned largely to the first category of the encompassing test (table 7). In the case of 20 days, this applies only to FV5.

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42 Moreover, an estimate adjusted for serial correlation does not show any perceptible change.

Table 9: Encompassing Test II (dependent variable: FV5)

Volatility	a	b	c	R^2	DW	t value (b=1)	t value (c=1)
60 days							
MEAN	-2.377 (1.500)	1.357 * (0.534)	0.053 (0.358)	0.617	1.392 U	0.669	-2.643 *
CALL	-2.168 (1.455)	1.277 * (0.511)	0.100 (0.347)	0.613	1.525 U	0.542	-2.593 *
KAPPA	-2.285 (1.503)	1.315 * (0.533)	0.082 (0.357)	0.611	1.422 U	0.590	-2.570 *
CALL&PUT	-2.053 (1.448)	1.226 * (0.505)	0.129 (0.345)	0.608	1.536 U	0.447	-2.524 *
PUT	-1.940 (1.441)	1.175 * (0.499)	0.157 (0.343)	0.602	1.546	0.351	-2.456 *
FED80	-1.740 (1.466)	1.141 * (0.534)	0.157 (0.372)	0.586	1.448 U	0.265	-2.265 *
FED60	-1.594 (1.475)	1.078 (0.539)	0.192 (0.378)	0.576	1.425 U	0.145	-2.138 *
40 days							
MEAN	0.864 (1.358)	0.827 (0.497)	-0.160 (0.335)	0.320	2.171	-0.349	-3.464 **
KAPPA	0.893 (1.352)	0.843 (0.511)	-0.176 (0.347)	0.318	2.188	-0.308	-3.395 **
FED60	1.150 (1.250)	0.792 (0.495)	-0.165 (0.350)	0.313	2.211	-0.421	-3.332 **
FED80	1.128 (1.270)	0.795 (0.503)	-0.165 (0.353)	0.312	2.204	-0.407	-3.301 **
CALL	1.021 (1.338)	0.823 (0.526)	-0.172 (0.361)	0.310	2.195	-0.336	-3.249 **
CALL&PUT	1.313 (1.338)	0.696 (0.531)	-0.095 (0.368)	0.286	2.168	-0.572	-2.978 **
PUT	1.599 (1.331)	0.569 (0.531)	-0.016 (0.372)	0.265	2.138	-0.812	-2.729 *
20 days							
PUT	1.631 (0.909)	0.698 (0.413)	-0.170 (0.377)	0.275	2.359	-0.731	-3.102 **
CALL&PUT	1.643 (0.924)	0.663 (0.423)	-0.136 (0.382)	0.262	2.304	-0.796	-2.970 **
CALL	1.665 (0.938)	0.619 (0.431)	-0.094 (0.386)	0.248	2.246	-0.884	-2.836 *
KAPPA	1.563 (0.976)	0.644 (0.459)	-0.112 (0.406)	0.244	2.227	-0.776	-2.741 *
MEAN	1.466 (1.024)	0.650 (0.482)	-0.108 (0.417)	0.239	2.214	-0.727	-2.659 *
FED80	1.846 (0.951)	0.434 (0.431)	0.052 (0.392)	0.208	2.135	-1.314	-2.419 *
FED60	1.901 (0.953)	0.374 (0.428)	0.101 (0.391)	0.198	2.101	-1.463	-2.298 *

R^2: adjusted coefficient of determination; DW: Durbin-Watson statistic; *,**: significant at the level of 5 or 1% (error level); U: DW is in the undetermined area; figures in brackets: standard errors of the estimates.

In addition, it is interesting to see how the (adjusted) coefficient of determination behaves for both FV and FV5 when the length of the tenor of the derivative is changed. Whereas, in the case of short maturities, variations of FV can be explained better than in the 60 days case, the opposite applies to FV5. It seems that the price volatility expected for the entire residual maturity increases in significance, the shorter the period until the expiry of the option is.[43]

2.2.3.4 Comparison of empirical analyses

The statement already made with respect to the simple qualitative and quantitative forecasts that the quality of the forecast increases as the residual maturity is reduced from three to about two calendar months, is only partly consistent with the results achieved by the Federal Reserve Bank of Atlanta, which carried out this kind of analysis on options on Standard & Poor's 500 index futures.[44] Considering the MSE, the Fed finds that, if all the data the from 1983 to end-1988 are used, a gradual reduction of the residual maturity from 57 to 38 or 17 trading days is accompanied by a lessening of the error. However, if the sample range is restricted, from 1983 to the stock market crash of 1987, the results are different both for the MSE and the MAE: in these cases, in fact, the prediction inaccuracy initially increases, with the result that (in contrast to the analysis carried out in the present study) the forecasts generated by the IV about two calendar months before the expiry of the option are less reliable than for τ=(60/252). It remains unclear whether the discrepancy is time-, country- or market-related.

As already mentioned, the procedure used for the regression analysis was chosen on the analogy of a study by the Bank of Japan (1995), which, inter alia, examined the information content of implied volatilities for Japanese bond futures. However, it uses only a single IV measure for its analysis, which more or less corresponds to the variable CALL&PUT[45]. Furthermore, in the case of regressions with FV as the dependent variable, quarterly data are used, which means that the residual maturities of the options are averaging more than three months (i. e. τ = 60/252). In the case of regressions with FV5 or

[43] If τ is reduced in excess of the values chosen here, FV and FV5 naturally converge gradually.

[44] The residual maturities examined by the Fed are 57, 38 and 17 trading days. They thus differ only slightly from the maturities chosen here.

[45] However, this measure incorporates the implied volatility of four options, instead of two.

IV as dependent variables, the data are collected monthly, with residual maturities varying accordingly.

In terms of information content, the most important results are as follows:

- The Japanese study concludes that more than 50 % of the variation in FV can be explained by implied volatilities (alone, or together with HV). These values are above the comparative values for the Bund futures.

- The adjusted coefficient of determination, for the explanation of FV5, at about 0.3, is distinctly below the R^2 in the 60 days case of the present study, but above that of the 40 days case. Since, owing to the monthly pattern of data collection, the residual maturity fluctuates approximately between one and three months, the difference from the Bund futures options should rather be regarded as negligible. Just as in the case of the 60 days forecast for Bund futures, the implied volatilities surprisingly appear to be superior to the historical ones for the explanation of FV5.

In Japan implied volatilities seem to yield a slightly better predictive performance for the future (long-term) volatility of bond futures than in the German market. However, the results are apparently relatively similar, at least for an international comparison. But a final assessment would necessitate identical studies in both markets.

3 Conclusion

The following is to be said regarding the use of historical and implied volatilities calculated from Bund futures options for predicting future price fluctuations:

- In the case of qualitative forecasts, i.e. forecasts of trends in future volatilities, implied volatilities are very successful in the Bund futures market. This applies to all three forecast horizons considered, with forecasts for a residual maturity of 20 trading days being somewhat more unreliable than those for longer forecast periods.

- When trying to predict future volatility qualitatively over the entire residual life of the option with the aid of implied volatilities, the accuracy of the forecast was increased when the tenor of the options was reduced from 60 to 40 trading days. However, the predictive power diminished if τ was lessened further. This statement applies both to simple quantitative forecasts and to the regression analysis. An exception to this is the encompassing test, where the highest coefficient of determination was shown for the case with the shortest forecast horizon.

- In addition, it is noteworthy that historical volatility can predict future volatility as measured over the entire residual maturity of an option more accurately than implied measures if there are no more than 20 trading days until the option expires. This is suggested both by the simple quantitative forecasts and by the regression analysis.

- Moreover, as expected, the accuracy of the forecast of FV increases if HV is used exclusively and the forecast horizon decreases. However, the intuitively persuasive explanation that shocks which have affected the development of HV also influence FV could not be fully corroborated. In fact, in the explanation of the volatility measured only on the five trading days after the determination of HV (i. e. FV5), which can be expected to be particularly affected by such shocks, the predictive quality of HV (and IV) is but low in the two shorter residual maturities.

- Although, for the longer residual maturities, the encompassing tests carried out to explain FV formally classify implied and historical volatilities in the same way, the t statistics of the regressions and the other tests suggest that the IV measures have a slight superiority. Of these, particularly CALL, KAPPA and MEAN are to be emphasised, which, viewed as a whole, are among the most reliable.

- If the proposition that the players in the option markets orient themselves only towards short-term trends is tested by means of a regression analysis, the evidence for this type of myopic behaviour is mixed. Thus, the fact that the IV measures, with FV5 as the dependent variable, are superior to historical volatilities for all maturities (as measured by the adjusted R^2) argues in favour of this kind of short-termism. In addition, the R^2 shown for $\tau=60/252$ and for the use, for example, of MEAN or CALL, at over 0.63, is far above the R^2 which is generated when FV is used as an endogenous variable. By contrast, just the opposite applies if $\tau=40/252$ and 20/252. In this case, the coefficient of determination suggests a tendency towards "longer-term" orientation.

- Taking due account of all analyses, therefore, the following could be shown by reference to the example of options on Bund futures: even though the accuracy of quantitative forecasts is not completely satisfactory if only implied and historical volatilities are considered, implied volatilities can be used successflully to predict the trend in the price volatility of the underlying asset with a high degree of reliability. This is an important result for central banks which in most cases are less interested in the precise extent of future price fluctuations than in the general trend in volatility at the long end of the bond market. Hence the use of IV measures for such forecasting purposes seems to be promising.[46]

It is obvious that the selection of IV measures in the present study does not exhaustively cover all possibilities. In fact, the Fed Atlanta's proposal, in particular, can be varied as often and as subtly as is deemed desirable. However, the suggestions tested here apparently cover the full range of the theoretically best-substantiated schemes. Any modifications of the weighting methods will very probably give rise to only slightly different results.

Viewed over all tests in the medium and long term, the measures CALL, MEAN and KAPPA appear to be the most appropriate ones, even if other measures score slightly higher in individual analyses. If, in addition, one considers the effort required for data processing, it seems reasonable to confine oneself, in the calculation of the implied volatilities, to the calculation of the call options that are at-the-money or just out-of-the-money, without having to expect serious quality losses. In the shape of this variable, the central bank therefore has not only an additional instrument for describing the market, but also an extra tool for forecasting purposes.

46 To some extent, this holds true for volatility traders as well. These option traders determine mainly by reference to the implied volatility whether options are overvalued or undervalued, and then implement a corresponding trading strategy in order to profit from potential imbalances or expected volatility changes.

IV. Implied probabilities

1 Procedures for determining implied probabilities

1.1 Implied probabilities in models of the Black-Scholes type

In chapter II, it was shown on the basis of the Black-Scholes model that it is possible to derive an option pricing formula by assuming a specific random process. In chapter III, this interrelationship was used in order to estimate, from given option premiums, the only unknown parameter (σ) of this random process, and to use it for forecasting the future volatility of the underlying asset. In addition, if σ is known, it is possible to make statements as to which probabilities are assigned under the Black-Scholes approach to particular values of the underlying asset on a specified day.[47] Once a probability distribution is known, many characteristic values, amongst other things, can be derived from it. The descriptive values make possible, in particular, statements on the uncertainty prevailing in the market - in a more detailed form than is possible using implied volatilities.[48] A consideration of probabilities may be more useful for many purposes than the exclusive concentration on a volatility variable which, by virtue of its construction, does not allow any direct statements to be made as to how the perceived probabilities will be distributed at particular points in time and, thus, how certain the market is about its own expectations.

For the price of the underlying asset, models of the Black-Scholes type necessarily generate implied probabilities which follow a log-normal distribution. This makes possible the calculation of implied probabilities for any day, and not only for the expiry date of an option. However, owing to the volatility smile, (see chapter 3), the problem arises that even within one single maturity class, different implied volatilities could be used to recover the implied probability density function. This, in turn, is inconsistent with the Black-Scholes model as it would produce different probability densities for the same underlying during a

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47 Strictly speaking, the Black-Scholes model can be used, owing to the use of continuous variables, only to make statements on the probability density for specific values. The probability for a defined interval enclosing this specific value can then be inferred from this.

48 It is even possible, on certain assumptions, to calculate an implied volatility from implied probabilities - as a special case, so to speak.

specific period of time. The resultant probability distributions would be contradictory. In this connection it would appear reasonable, of course, to fall back on one of the volatility measures which best forecasts the future volatility, such as one of the variables CALL, KAPPA or MEAN from the previous chapter.

However, it should be borne in mind that, although the empirical analysis in the above chapter demonstrated the reliability of these IV measures with regard to forecasting the direction in which future volatility will move, quantitative forecasts had to be classified as imprecise. This may be because reality is described but imprecisely by the assumption of the geometrical Brownian motion process - and thus the log-normal distribution of the underlying asset (for any given point of time). It is possible, for example, that the price of the underlying asset jumps from time to time (jump diffusion process) or that the true distribution of the underlying asset has a changeable standard deviation. The price of the underlying contract may then be determined by a mixture of normal distributions. It is also possible that the price of the underlying asset does not follow a log-normal distribution, but simply another type of distribution.[49] This is why it seems more advantageous to choose a flexible approach which does not require any *ex ante* distribution assumption for calculating the implied probabilities.

1.2 Distribution-free approaches used hitherto for calculating implied probabilities

In the relevant literature, various proposals for calculating implied probabilities have already been elaborated. Malz[50], for instance, allows the price of the underlying asset to follow a jump diffusion process, whereas Melick and Thomas[51] proceed from the assumption that the probability density of the underlying asset can be calculated from a mixture of up to three log-normal distributions. Both approaches include the Black-Scholes model as a special case, but also allow the discovery of other probability distributions.

The main advantage of these models is that they make do with only a few data, since they assume a certain structure in each case. If this structure is correct, the respective approach generates precise statements on the implied probabilities, even with a limited data input. If

49 See, for example, GEMMILL (1993), page 113.
50 See MALZ (1994).
51 See MELICK, THOMAS (1994).

the assumed random processes or distribution functions reflect reality only insufficiently, however, the results generated are not very reliable.

If the *ex ante* knowledge is not sufficient to assume a specific structure, less restrictive methods have to be adopted. Such a method was developed by Breeden and Litzenberger (1978), and is ultimately based on equation **(2)** introduced in the second chapter, according to which the price of the European-style call option in a risk-neutral world follows from the (discounted) expected pay-offs:

(7) $$C = e^{-rt} \int_{-\infty}^{+\infty} w(F_T)\max(0, F_T - K)dF_T$$.[52]

If a daily margining of the options is carried out, the formula can be simplified to:

(7') $$C = \int_{-\infty}^{+\infty} w(F_T)\max(0, F_T - K)dF_T$$.

As in the case of implied volatilities, the probability density is calculated from this equation for known C and $\max(0, F_T - K)$. If one determines the second-order partial derivative of the call price with respect to the strike price, taking due account of the corresponding rules, one directly obtains the probability density **for F_T=K**:

(8) $$C_K = -\int_{K}^{+\infty} w(F_T)dF_T$$,

(9) $$C_{KK} = w(K)$$.

A probability is derived by calculating an integral which includes K, for example

(10) $$p(K_i^*) \overset{!}{=} p(K_i - 0{,}5\kappa \le F_T \le K_i + 0{,}5\kappa) = \int_{K_i - 0{,}5\kappa}^{K_i + 0{,}5\kappa} w(F_T)dF_T$$.[53]

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[52] Reminder: F_T denotes the variable: "Price of the underlying asset (F) on the expiry day T". T is thus a constant, which is of particular importance for generating the derivatives.

[53] The approach can also be carried out with puts. Owing to put-call parity, the results are identical when using (de facto) European-style options.

In the practical application of this approach, there arises the problem of the only finite number of strike prices. Whereas the deduction assumes a variable C which is continuous in K, only a limited number of options are traded on the exchanges for each maturity class. This problem can be overcome in two different ways. The first approach was chosen by Shimko (1991). He generates the missing call prices by fitting a parabola to the volatility smile and determining a σ for all K. Subsequently, he calculates, by means of the appropriate Black-Scholes formula, the call prices at whatever close intervals he chooses;[54] this makes possible the numerical calculation of the first and second partial derivative of C with respect to K. However, the results of this procedure very largely depend on the interpolation technique used. In addition, a mispricing of the synthetically calculated premiums is not ruled out *ex ante*. This problem becomes acute, in particular, if options are included in the volatility smile that are deep in-the-money or out-of-the-money. Their prices should mainly be composed of their intrinsic values, but this is not guaranteed if the premium of an unobservable (i.e. synthetic) option is calculated via the "detour" of a "fitted smile". Owing to the failure to take account of all the arbitrage restrictions applicable to option prices, important available information is disregarded. In addition, the interpolation feigns a degree of security concerning the knowledge of unobservable call prices, which is illusory.

A second possible approach was proposed by Breeden and Litzenberger (1978) themselves.[55] It is based on a simple approximation of C_{KK} by means of the second-order difference quotient. If the difference between adjacent strike prices is always ΔK, the probability density at a specific strike K_i[56] thus follows from:

$$(11) \qquad w(K_i) \approx \frac{C_{i-1} - 2C_i + C_{i+1}}{\Delta K^2} \quad .$$

The probability that, on the expiry date of the option, the value of the underlying asset will be within an interval between K_i-0.5κ and K_i+0.5κ - for notational simplicity written as $p(K_i{*})$ - is therefore:

54 However, this does not mean that the Black-Scholes formula is accepted as valid; it serves only as an iteration aid.

55 For an application which constitutes a parallel development, in spite of the time-lag vis-à-vis the original paper, see PINKAVA (1994).

56 The running index i=1, ..., n assumes the value 1 for the most expensive call option, i.e. the call option with the lowest strike price.

(12) $p(K_i^*) \approx w(K_i) \cdot \kappa$

For κ = ΔK, then, the following applies:

(13) $p(K_i^*) \approx \dfrac{C_{i-1} - 2C_i + C_{i+1}}{\Delta K}$

For the calculation, therefore, only the prices of three call options are necessary, the strike prices of which are ΔK units apart. To generate a complete probability distribution, as large a range as possible of options of the same maturity class with different strike prices is needed.

Both approaches have one disadvantage in common: if the range of observable strikes is not wide enough to cover the entire spectrum of possible realisations, the sum total of the probabilities shown will be smaller than unity.[57] Although it is of course possible to determine the missing "probability mass" (which is simply unity minus the sum of the already calculated probabilities), it is not always discernible in the approaches described above how much of the missing probability is attributable to the upper edge of the histogram and how much is attributable to the lower edge, which can be a serious drawback.

1.3 A new approach

In the following section, a newly developed approach is to be presented which, like the methods described above, dispenses with restrictive assumptions, and, in addition, includes information on probabilities at the edges of the probability distribution even if there are not enough strikes to cover the whole range of the distribution. This is achieved by inferring not the probability *density* but rather the implied probability *distribution* (ipd) from the option prices. This is done because call premiums, like distribution functions, only contain information on *all* the pay-off probabilities *above* the respective strike price - and not on *some* pay-off probabilities in the *vincinity* of or precisely at specific strikes, which is what the recovery of the probability density function would generate.

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[57] For the discrete approach, this implies that more than their intrinsic value is paid for the two extreme (observable) in and out-of-the-money options.

The distribution is obtained, marked with a minus sign, from the first derivative of the call premium with respect to the strike price (see equation (8)).

$$(8') \; C_K = -\int_K^{+\infty} w(F_T)dF_T = -p(K \le F_T \le \infty) = -p(F_T \ge K) \qquad \text{or}$$

$$(8'') \quad -C_K = p(F_T \ge K) \quad .$$

The first derivative, therefore, contains information on the size of the probability, when F is greater than or equal to the given strike price K.[58] In order to avoid the problem of mispricing and the selection of an iteration technique, which may affect the result, the first derivative is to be approximated discretely by using the first-order difference quotient. The premiums used should stem from options with adjacent strikes, whose premiums (for increasing strike prices) decline monotonously and are convex.[59] If the strikes are ΔK units apart, we have

$$(14) \quad p(F_T \ge K_i) \approx \frac{C_{i-1} - C_{i+1}}{2 \cdot \Delta K} \quad .$$

The (perceived) probability $p(K_i \le F_T \le K_{i+1})$, with which, on the expiry day of the option, F is between the strike prices K_i and K_{i+1} - for notational simplicity referred to as $p(K_i)$ - therefore follows from:

$$(15) \; p(K_i) = p(F_T \ge K_i) - p(F_T \ge K_{i+1}) \approx \frac{[C_{i-1} - C_{i+1}] - [C_i - C_{i+2}]}{2 \cdot \Delta K}$$

A comparison of the probabilities $p(K_i)$ and $p(K_i^*)$ (equations (15), (10) and (13)) reveals a difference: $p(K_i^*)$ describes the probability of F_T being in an interval around the strike K_i and not between two strike prices. One of the advantages of the method presented here is that, for the calculation of probabilities by means of equation (14), $p(F_T < K_i)$ is always determined as well. Even if not enough option prices are known, statements can thus be made, at least on the probability with which F_T-values are smaller than K_2 and larger than

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58 Strictly speaking, this (negative) first derivative is numerically identical to 1 less the distribution function.

59 To put it more precisely: The option premiums can be linear or left-handed curved.

K_{n-1}. This cannot be done as easily, or not at all, by the Breeden-Litzenberger procedure described in the last section.

2 Properties of the implied probabilities

Owing to the arbitrage which would otherwise be possible, option premiums decline monotonously and are linear or convex. This behaviour guarantees a value of $p(K_i) \geq 0$. Another way of explaining why the probabilities generated are non-negative is that the numerator in the fraction of equation **(15)** can also be regarded as the difference between two call spreads. The first, compared with the second, is more likely to generate a pay-off, and therefore should not cost less. If all arbitrage possibilities are exploited, p cannot be negative.

If $p(K_i) \geq 0$ is violated, the premiums were recorded either incorrectly or imprecisely. This may simply be attributable to the fact that not enough decimal places are used in the quotation of the option. However, it is more likely that the option prices were not recorded simultaneously. If the price data are not collected simultaneously, an erroneous calculation of probabilities may ensue. This is why it seems appropriate to include only the closing prices of the options concerned. It may also be that the prices quoted by a stock exchange or by market makers are mere indicators. But even in this case, the deviations from the "true" prices will tend to be low, since the profit of the entities mentioned depends not least on satisfied customers who, in particular, therefore have to be provided with reliable price data.

Another, fundamental, property of probabilities is that they have to add up to one. If there are n options in a maturity class, the addition of all probabilities yields the following:

$$\textbf{(16)} \quad \sum_{i=2}^{n-2} p(K_i) \approx \frac{C_1 - C_3 + (C_{n-2} - C_n)}{2 \cdot \Delta K} .$$

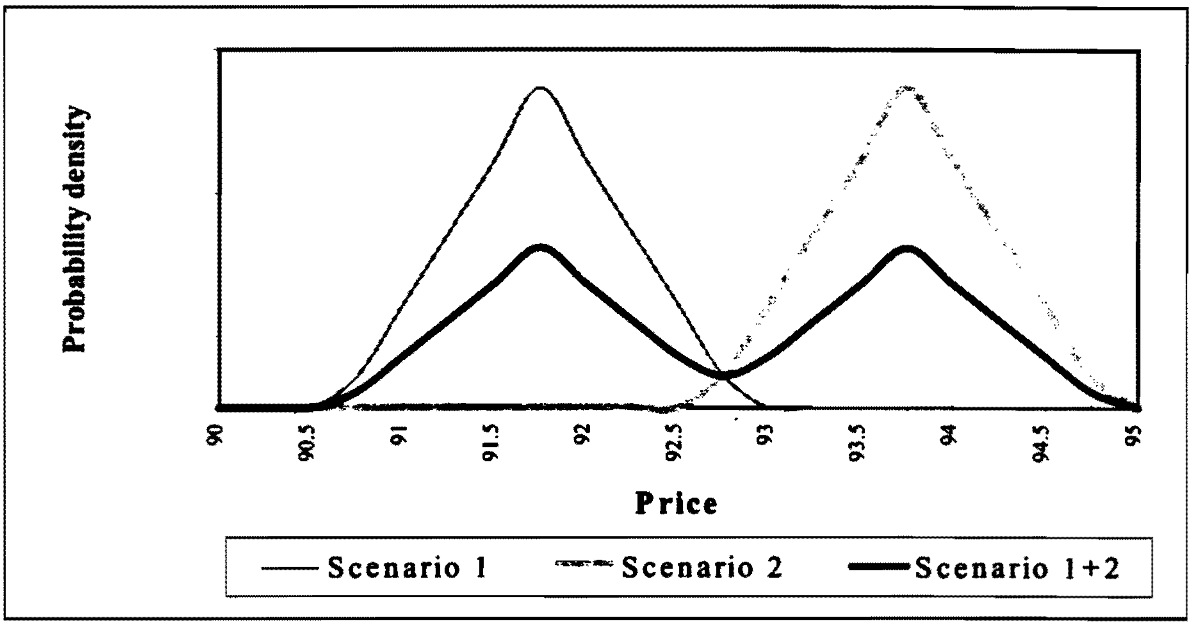
If the range covered by the n options is wide enough, the options at the edges are traded in accordance with their intrinsic value. It follows from this that

$$\textbf{(16')} \quad \sum_{i=2}^{n-2} p(K_i) \approx \frac{(F - K_1) - (F - K_3) + (0 - 0)}{2\Delta K} = \frac{2\Delta K}{2\Delta K} = 1 .$$

The variable calculated by means of the approximation of the first derivative thus conforms to the basic properties which a probability distribution must satisfy. As a consequence, the data generated in this way can also be used for further calculations, such as ascertaining the expected value. If, as in the third chapter, options on futures are used, the expected value would have to be identical with the current futures price.[60]

Needless to say, the information content of a complete probability distribution for possible futures values exceeds that of a point estimator, such as the current futures or forward price, and is thus superior to it. For instance, the calculation of simple dispersion measures (variance of the distribution or interquartile range) is sufficient to provide a yardstick of the uncertainty prevailing in the market that is associated with the implied forecast. If, for example, the uncertainty concerning future developments increases, this is not necessarily reflected at once in the expected value (point estimator). However, the range of realisations deemed to be probable is likely to increase. This, in turn, will presumably be reflected in changed dispersion measures.[61]

Figure 4: Multi-modal probability density of two expected scenarios



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[60] In accordance with theoretical considerations, in a risk-neutral world the expectation value of a forthcoming futures price equals its current value. The price of futures whose purchase or sale effectively costs nothing is therefore a martingale.

[61] In addition, moments of higher order can be calculated as well.

Furthermore, it may be worthwhile to plot the determined probabilities in a histogram. In this way, it is even possible to detect a bi-modal or multi-modal distribution. Such a pattern, which - technically speaking - results from the varying convexity of the option premiums, is likely, in particular, if market players proceed from two or more scenarios. Thus, the view might prevail that either a large upward price movement or a downward shift is imminent. Figure 4 illustrates such a case - by reference to probability densities: The market "assumes" two scenarios of identical probability, the modes of which are at 91.75 and 93.75, respectively. If one pools the probabilities, a bi-modal function emerges. If such bi-modal or even multi-modal functions exist, focussing on expected values or other point estimators involves the danger that the picture one gets of market expectations is wrong.

One circumstance requires special explanation: the implied probabilities were determined on the assumption that economic agents are risk-neutral.[62] However, a market player who was averse to risk would be prepared to pay the expected value of a call option plus a risk premium. Hence one would determine not the probability expected by market players, but only a distorted estimate. In such circumstances, the utility function of the market players would have to be known in order to generate undistorted estimates. However, Rubinstein (1994) shows by means of an example that, even if different risk premiums are assumed, the risk-neutral probabilities are a close approximation to the probabilities assumed by the market. An indication of the similarity of the probability measures can also be found in the present paper: in the third chapter, no systematic forecast error could be found for any volatility measure or for any of the three residual maturities examined, which would have hinted at the existence of risk premiums. Even where the recovered probabilities are not wholly identical to those expected by the market, there is much evidence suggesting that the difference is only slight.

3 Empirical analysis

The approach described above is applicable to all options of the (de facto, at least) European type, with a large number of strike prices for each maturity class being conducive to the quality of the calculation. In the following section, the approach described above is

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62 This is the case in all approaches and, consequently, also applies to the values calculated by means of a formula of the Black-Scholes type.

applied to the premiums of Bund futures and Euro-mark futures options. In both cases, the problems arising in the practical application of the method are to be evaluated.

3.1 Options on Bund futures

3.1.1 Method

As in the third chapter, the pertinent options of the LIFFE are used, particularly because of their liquidity, the margining and the multitude of strike prices in the individual maturity classes. In order to give an idea of how the calculations are effected, table 10 contains the data recorded by the exchange (columns 1-4) and the calculations carried out.

In columns 5 and 6, the probability distribution and the probabilities of the individual classes are estimated by means of equations (14) and (15), respectively. Owing to the use of first-order difference quotients, the value of the probability distribution can be estimated only from the second class (89.5-90.0) onwards. It amounts to 0.98. It can be inferred from this that the missing two percentage points are to be ascribed to lower Bund futures values. For lack of better information, they are assigned to the smallest class.

Analogously, it is possible that the strike prices quoted at the other edge are insufficiently spread out as well. The value of the distribution function shown in this case and the corresponding probability of the last class would then be greater than the true value. This is why the probabilities of the classes at the edges and all the measures which incorporate them - such as the expected value, the range of distribution and the variance - are to be interpreted with caution. Only if there is at least one class each with the values one or zero for the distribution function can it be guaranteed that the probabilities of the marginal classes are represented correctly. Failing this, this cannot be assured.[63]

A possible way of "testing" the reliability of the recovered probabilities is to determine the "empirical" expected value and to compare it with its "theoretical" equivalent - the futures

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[63] For example, it is not apparent from table 10 whether the probability of 1 % ascribed to the class 104 - 104.5 coincides really precisely with this interval, or whether part of it should be ascribed to higher futures values. This question can be answered with certainty only if there were a next class and the distribution function for this interval assumed a value of zero. However, by the Breeden-Litzenberger procedure one could only perceive how much of the overall probability mass is missing, but not necessarily which edge has to be ascribed how much of that mass.

price actually traded. For this purpose, the product of the class mean and the associated probabilities was calculated and added up in column (8). The empirical expected value determined in this way from the closing prices of the options, amounting to 96.44, differs only slightly from the settlement price of the Bund future on the same day, which was actually two basis points lower.

Table 10: Calculation of implied probabilities from the premium of Bund future call options

(1) Date	(2) Expiry Month	(3) C	(4) K	(5) Distribution	(6) p(K) [%]	(7) class middle	(6*7)
04/03/94	Jun 94	7.44	89		2.0	89.25	1.79
04/03/94	Jun 94	6.95	89.5	0.98	1.0	89.75	0.90
04/03/94	Jun 94	6.46	90	0.97	2.0	90.25	1.80
04/03/94	Jun 94	5.98	90.5	0.95	1.0	90.75	0.91
04/03/94	Jun 94	5.51	91	0.94	2.0	91.25	1.83
04/03/94	Jun 94	5.04	91.5	0.92	2.0	91.75	1.83
04/03/94	Jun 94	4.59	92	0.9	2.0	92.25	1.85
04/03/94	Jun 94	4.14	92.5	0.88	1.0	92.75	0.93
04/03/94	Jun 94	3.71	93	0.87	0.0	93.25	0.00
04/03/94	Jun 94	3.27	93.5	0.87	6.0	93.75	5.63
04/03/94	Jun 94	2.84	94	0.81	9.0	94.25	8.48
04/03/94	Jun 94	2.46	94.5	0.72	6.0	94.75	5.68
04/03/94	Jun 94	2.12	95	0.66	3.0	95.25	2.86
04/03/94	Jun 94	1.8	95.5	0.63	6.0	95.75	5.75
04/03/94	Jun 94	1.49	96	0.57	8.0	96.25	7.70
04/03/94	Jun 94	1.23	96.5	0.49	6.0	96.75	5.81
04/03/94	Jun 94	1	97	0.43	6.0	97.25	5.83
04/03/94	Jun 94	0.8	97.5	0.37	8.0	97.75	7.82
04/03/94	Jun 94	0.63	98	0.29	3.0	98.25	2.95
04/03/94	Jun 94	0.51	98.5	0.26	4.0	98.75	3.95
04/03/94	Jun 94	0.37	99	0.22	5.0	99.25	4.96
04/03/94	Jun 94	0.29	99.5	0.17	3.0	99.75	2.99
04/03/94	Jun 94	0.2	100	0.14	4.0	100.25	4.01
04/03/94	Jun 94	0.15	100.5	0.1	2.0	100.75	2.02
04/03/94	Jun 94	0.1	101	0.08	3.0	101.25	3.04
04/03/94	Jun 94	0.07	101.5	0.05	2.0	101.75	2.04
04/03/94	Jun 94	0.05	102	0.03	1.0	102.25	1.02
04/03/94	Jun 94	0.04	102.5	0.02	0.0	102.75	0.00
04/03/94	Jun 94	0.03	103	0.02	0.0	103.25	0.00
04/03/94	Jun 94	0.02	103.5	0.02	1.0	103.75	1.04
04/03/94	Jun 94	0.01	104	0.01	1.0	104.25	1.04
04/03/94	Jun 94	0.01	104.5				
Sum total					100.0		96.44
Memorandum item: futures price of the June contract on 04/03/1994							96.42

3.1.2 Changes in the probability distribution over time

Implied probabilities constitute an instrument which helps to describe the financial markets concerned. In particular, they make it possible to trace the development of the expectations of market players and the associated uncertainty. Since a representation of the probabilities or histograms generated in an initial step appears too complex for monitoring a market over long periods of time, a simplified depiction has to be made, for example, by means of a dispersion parameter. The usual measures for that are the standard deviation, the average deviation, the range[64] and the interquartile range. The standard deviation and the average deviation have the disadvantage that, by definition, they are not capable of representing asymmetrical dislocations of distributions. In addition, the classes at the edges also enter into both dispersion parameters. The same applies to the range of the distribution. This may pose problems if the spectrum of existing strike prices is not sufficient and the premiums of the extreme options, i.e. the prices of the cheapest and the most expensive options, overshoot their intrinsic value. The interquartile range, which defines the 50% confidence interval and is measured as the distance between the 75 % quantile and the 25 % quantile, is not affected by this problem.[65] Of course, other confidence intervals are conceivable as well. The choice has to depend on the extent to which "extreme" expectations are to influence the indicator. For example, the difference between the 10 % and the 90 % quantile would represent an area which is more likely to enclose the value of the underlying asset of the option. However, it would reflect the "mainstream" expectations less precisely than the interquartile range, which, after all, comprises a "probability mass" of 50 %.[66] The mainstream as such is naturally represented by the expected value of the market, which is approximated by the current futures price. The latter is to be represented together with the 50 % confidence interval, in order to examine whether the indicator "implied probabilities" can reveal more information than a simple point estimator.

Since the probabilities and their distribution are available only by class, a uniform distribution was assumed to prevail within the classes. This is, of course, a simplifying

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64 The difference between the extreme values of a distribution.

65 In this paper, a p% quantile (F_p) is defined as the possible futures value to which the following applies: $p(F_T \geq F_{T,p})=p\%$. As an illustration, an example using the values from table 10: The market estimate on 04/03/94 is as follows: With a probability of 75 %, the futures price of the June contract on the expiry date of the option will be above 94.33. This value is therefore the 75 % quantile or the 75% threshold.

66 The more extreme (more on the outside) the chosen quantile is, the more the "edge class problem" will become relevant again.

assumption, and the quantiles calculated should therefore not be regarded as a hundred percent precise, but, instead, as plausible approximations.[67]

The approach and the development of the interquartile range are to be illustrated by means of an example. Preferably, a period should be chosen in which a special event occurred. The publication of major monetary policy measures or the announcement of other relevant data seem eminently suitable for this purpose.

An illustrative example is the publication on March 2, 1994 of the surprisingly high rate of change of the money stock aggregate M3 in January 1994. The initially published annualised increase of 20.6 %, given a target corridor of 4-6 % and a repeated overshooting of the money stock target in the previous year, was almost bound to unsettle market players. After all, the Bundesbank is constantly referring to the long-term interrelationship between an excessive liquidity supply in the economy and the inflation rate.[68] As a result, monetary trends drew attention to the increased danger of a dramatic rise in prices - a development for which investors at the long end of the bond market want to be recompensed by a higher yield, and which therefore is accompanied by lower bond prices. In addition, market players could not know how the Bundesbank would respond. Would it be possible to continue the policy of "tiny steps", or would monetary policy have to be tightened? And - which is at least of the same importance - would the German central bank be successful in keeping price buoyany under control? After all, adopting, for example, a contractionary monetary policy stance may depress the prices of bonds. However, if it proves possible to counteract inflation credibly by this measure, long-term bonds, in particular, can benefit from it. In brief: a useful indicator should show an increased level of uncertainty with regard to market expectations during this period.

Figure 5 shows the development of the futures price (June 94 contract) and the quartiles (25 % and 75 % threshold), which surround the futures like a corridor of flexible width, in the period from February 1 to March 31, 1994. In addition, the interquartile range is calculated from the variables represented. In order to make it clear to what extent a change in the dispersion parameter is attributable to the change in the "upper" or the "lower" side

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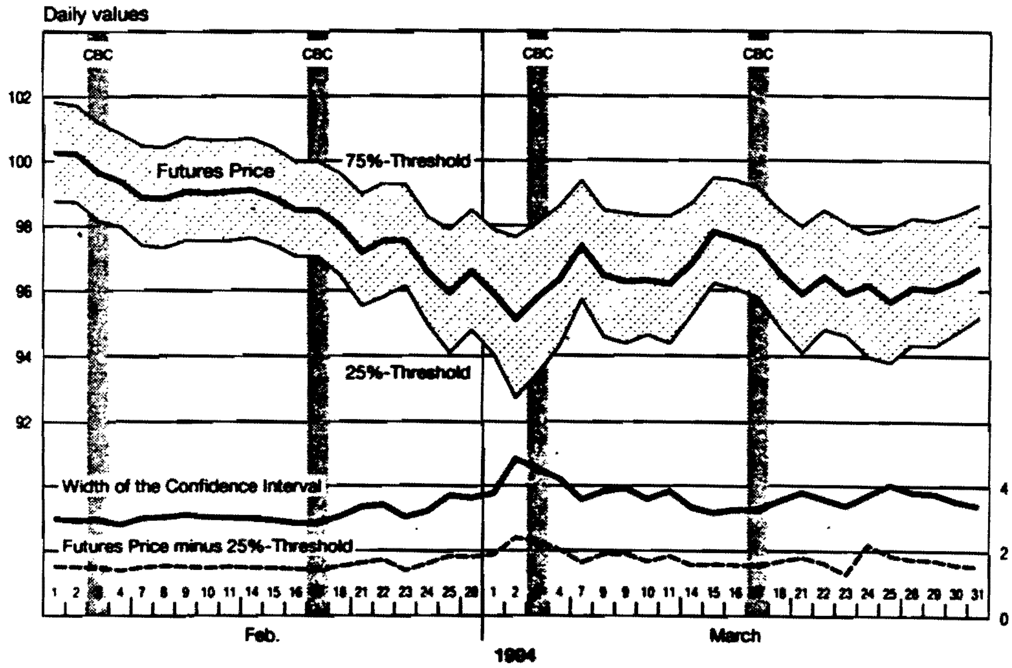
67 The similarity of the theoretically and empirically determined expected values of both Bund futures and Euromark futures, suggests that the assumption of a uniform distribution within the classes poses no problems.

68 See, for example, DEUTSCHE BUNDESBANK (1992).

of the corridor, the difference from the futures price and the 75 % quantile is shown as well.

Figure 5: Price and confidence interval of the Bund future



* On the basis of the Bund future and options on the June 1994 contract. Source: LIFFE, own calculations. "CBC" indicates that on these days a Central Bank Council meeting took place.

Between February 1 and the middle of that month, the quantiles moved in line with the futures price, which declined initially and then moved sideways. Subsequently, the price again tended to decline; the spread between the quartiles and futures also increased slightly. At the end of February, at a time when normally the preliminary money stock figures for the previous month are published, the width of the confidence interval widened to just under four points. When the M3 data were finally published on March 2, the futures market collapsed further, and the difference between the 25 % and 75 % threshold shot upwards by almost 100 basis points. Since the impact of the M3 news on both interest-rate decisions and the outlook for inflation was not clear to market participants, the uncertainty in the market did not decline to the level that had prevailed prior to the data release. Hence, market players perceived that the possibility of larger price movements occuring in the future had risen.

Figure 5, which in a simplified form shows the change in the implied probability distributions of Bund futures options, naturally does not indicate precisely which monetary policy implications were expected by market players at that time. However, in many cases an indirect conclusion is possible. Thus, the reduction in the discount rate on February 17 is not reflected in a change in the indicator, suggesting that such a step was expected, at least in general terms, even if the timing might have come as a surprise. In order to find out more exactly what was expected with regard to the timing and scope of monetary policy measures, options on short-term interest rate contracts have to be examined as well.

3.2 Options on Euromark futures

Before considering confidence intervals calculated from options on short-term interest rate contracts, a specimen calculation is to be carried out, in order to clarify the differences between an application of the new approach presented here on the option markets involved in this study.

3.2.1 Method

Just as in the previous empirical analyses, the data used were provided by LIFFE, viz. the prices of options on Euromark futures. The Euromark future is a forward contract traded on LIFFE on a three-month interest rate transaction with the nominal value of DM 1 million. The interest rate x%, which is paid on the nominal value, is calculated from the futures price. According to the formula "x=100-F", the implied interest rate for a futures price of 94.50 comes to x=5.50 %.[69]

The premiums of the options on these futures can be used, as before, to calculate the implied probabilities. In contrast to the previously considered derivatives it is immaterial, in the procedure applied here, owing to the linear relationship of the two variables, whether the futures price or the implied (forward) interest rate is used as the underlying asset (see tables 11 and 12). For that reason, and because they are easier to interpret, the probabilities for interest-rate intervals, rather than price intervals, will be indicated below.[70]

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69 As the contract is settled in cash, there is no delivery. For more details, see LIFFE (1994), page 21 ff.

70 Owing to the inverse relationship between the price and the interest rate, the quantiles are now also defined differently. Thus, in the example given in table 11, the implied interest rate will be **below** the

Compared with the above implied distributions, the following difference is conspicuous: the range of strikes is so wide that all the realisation possibilities deemed likely are covered. In the example (table 12), the spectrum ranges from a three-month interest rate of 3.50 % to 7.50 %. However, only the area of 4.25 % (!) to 6.75 % is considered possible, as can be seen from the distribution function (column 5). Not least, knowledge of the complete distribution function permits the precise calculation of the empirical expected value, which in the example chosen (but not only there) is identical to its theoretical value, the futures price. In addition, owing to the wide range of strike prices quoted by LIFFE, the "edge class problem" does not arise for the options considered here, which facilitates the reliable calculation of moments of higher order and also of dispersion measures other than the interquartile range. Nevertheless, for the above-mentioned reasons, we shall stick to the latter.

3.2.2 Changes in the probability distribution over time

As in section 3.1.2, in the following section we shall consider the movement of probability distributions over time. For this purpose, we shall once again contemplate the months of February and March 1994 more closely (figure 6).

If o· · considers the movement of the indicator more closely, it is easy to trace the pattern expectations in the money market. At the beginning of February 1994, the Federal Reserve's tightening of the monetary reins surprised the money markets and temporarily increased uncertainty in the Euromark market. The width of the confidence interval - the interquartile range - increased from 50 bp to 55 bp. If one considers the absolute values of the quantiles, one sees that the market "assumed", with a probability of 50 %, that the interest rate for three-month funds[71] on the expiry date of the option would be between 4.95-5.50 %.

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interest rate of 5.25 % with a probability of 26 % when the option expires. Nevertheless we can continue to make the statement that the according to market expectations futures price will be above the futures price of 94.75 % with a probability of 26 % when the option expires.

[71] Strictly speaking, this is the interest rate implicitly included in the June contract of the Euromark future. Since, however, options and futures expire at the same time, the implied and actual interest rates on the expiry date are almost identical.

Table 11: Calculation of implied probabilities from the premiums of Euromark future call options (underlying = futures price)

(1) Date	(2) Expiry month	(3) C	(4) K	(5) Distribution	(6) p(K) [%]	(7) Class middle	(6*7)
04/03/94	Jun 94	2	92.50		0	92.625	0
04/03/94	Jun 94	1.75	92.75	1	0.0	92.875	0
04/03/94	Jun 94	1.5	93.00	1	0.0	93.125	0
04/03/94	Jun 94	1.25	93.25	1	2.0	93.375	1.8675
04/03/94	Jun 94	1	93.50	0.98	2.0	93.625	1.8725
04/03/94	Jun 94	0.76	93.75	0.96	8.0	93.875	7.51
04/03/94	Jun 94	0.52	94.00	0.88	16.0	94.125	15.06
04/03/94	Jun 94	0.32	94.25	0.72	22.0	94.375	20.7625
04/03/94	Jun 94	0.16	94.50	0.5	24.0	94.625	22.71
04/03/94	Jun 94	0.07	94.75	0.26	14.0	94.875	13.2825
04/03/94	Jun 94	0.03	95.00	0.12	6.0	95.125	5.7075
04/03/94	Jun 94	0.01	95.25	0.06	4.0	95.375	3.815
04/03/94	Jun 94	0	95.50	0.02	2.0	95.625	1.9125
04/03/94	Jun 94	0	95.75	0	0.0	95.875	0
04/03/94	Jun 94	0	96.00	0	0.0	96.125	0
04/03/94	Jun 94	0	96.25	0	0.0	96.375	0
04/03/94	Jun 94	0	96.50				
Sum total:					100		94.5
Memorandum item: futures price on 04/03/1994 (June future, settlement):							94.5

Table 12: Calculation of implied probabilities from the premiums of Euromark future call options (underlying = implied interest rate)

(1) Date	(2) Expiry month	(3) C	(4) K	(5) Distribution	(6) p(K) [%]	(7) Class middle	(6*7)
04/03/94	Jun 94	2	7.50		0	7.375	0
04/03/94	Jun 94	1.75	7.25	1	0.0	7.125	0
04/03/94	Jun 94	1.5	7.00	1	0.0	6.875	0
04/03/94	Jun 94	1.25	6.75	1	2.0	6.625	0.1325
04/03/94	Jun 94	1	6.50	0.98	2.0	6.375	0.1275
04/03/94	Jun 94	0.76	6.25	0.96	8.0	6.125	0.49
04/03/94	Jun 94	0.52	6.00	0.88	16.0	5.875	0.94
04/03/94	Jun 94	0.32	5.75	0.72	22.0	5.625	1.2375
04/03/94	Jun 94	0.16	5.50	0.5	24.0	5.375	1.29
04/03/94	Jun 94	0.07	5.25	0.26	14.0	5.125	0.7175
04/03/94	Jun 94	0.03	5.00	0.12	6.0	4.875	0.2925
04/03/94	Jun 94	0.01	4.75	0.06	4.0	4.625	0.185
04/03/94	Jun 94	0	4.50	0.02	2.0	4.375	0.0875
04/03/94	Jun 94	0	4.25	0	0.0	4.125	0
04/03/94	Jun 94	0	4.00	0	0.0	3.875	0
04/03/94	Jun 94	0	3.75	0	0.0	3.625	0
04/03/94	Jun 94	0	3.50				
Sum total:					100		5.5
Memorandum item: implied three-month interest rate on 04/03/1994 (June future):							5.5

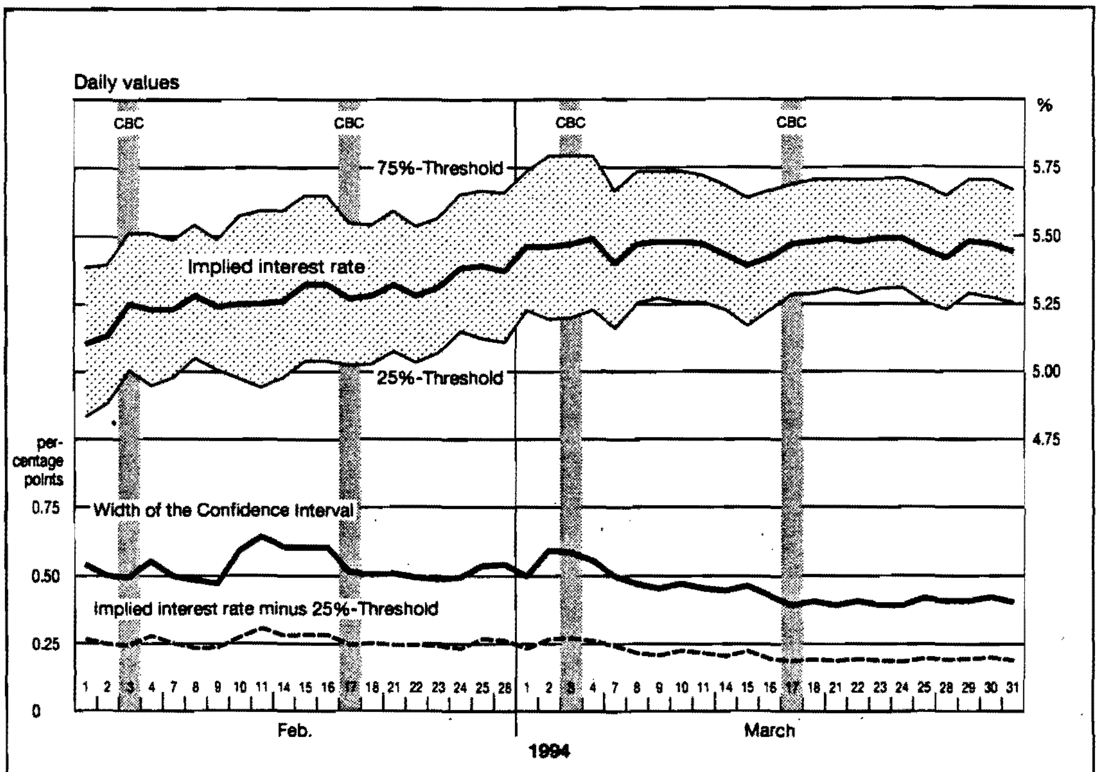
Subsequently, markets calmed down until, in the run-up to the Central Bank Council meeting on February 17, uncertainty about the Bundesbank's interest-rate decision finally increased again. The indicator chosen shows a maximum difference of 0.65 percentage points. Only when, on February 17, the decision to lower the discount rate by one-half of a percentage point was taken and made public, did the uncertainty in the market slacken for a time.

It was rekindled by the belated publication of the M3 figure for January. It was only when the Bundesbank's statement that the figure published for January was distorted by special factors and by the features of annualisation, and that the Central Bank Council would not thereby be forced immediately to tighten monetary policy, was accepted by the market that the dispersion of expectations decreased progessively, actually falling below the level reached at the beginning of February.

The charts may bear out theoretical notions that the money and bond markets respond differently to identical events. Whereas the interest rate reduction by the Bundesbank on February 17, 1994 had apparently already been discounted in bond prices and (bond future) option prices, the interquartile range calculated from Euromark futures options shows, in addition, that the question of the extent and timing of the interest rate measure was strongly affecting the money market at that time.

If one simultaneously considers the interquartile ranges of the Euromark and the Bund futures, it transpires that, in the money market, unlike the bond market, uncertainty after the first week of March was below the level reached at the beginning of February. This is another sign indicating that the prices of long-dated government bonds are not influenced as directly by monetary policy decisions as are the rates for three-month funds. The uncertainty detected in the case of the Bund future probably tends to reflect, rather, the (apparently) increased inflationary pressure due to monetary developments.

Figure 6: Implied interest rate and confidence interval of the Euromark future



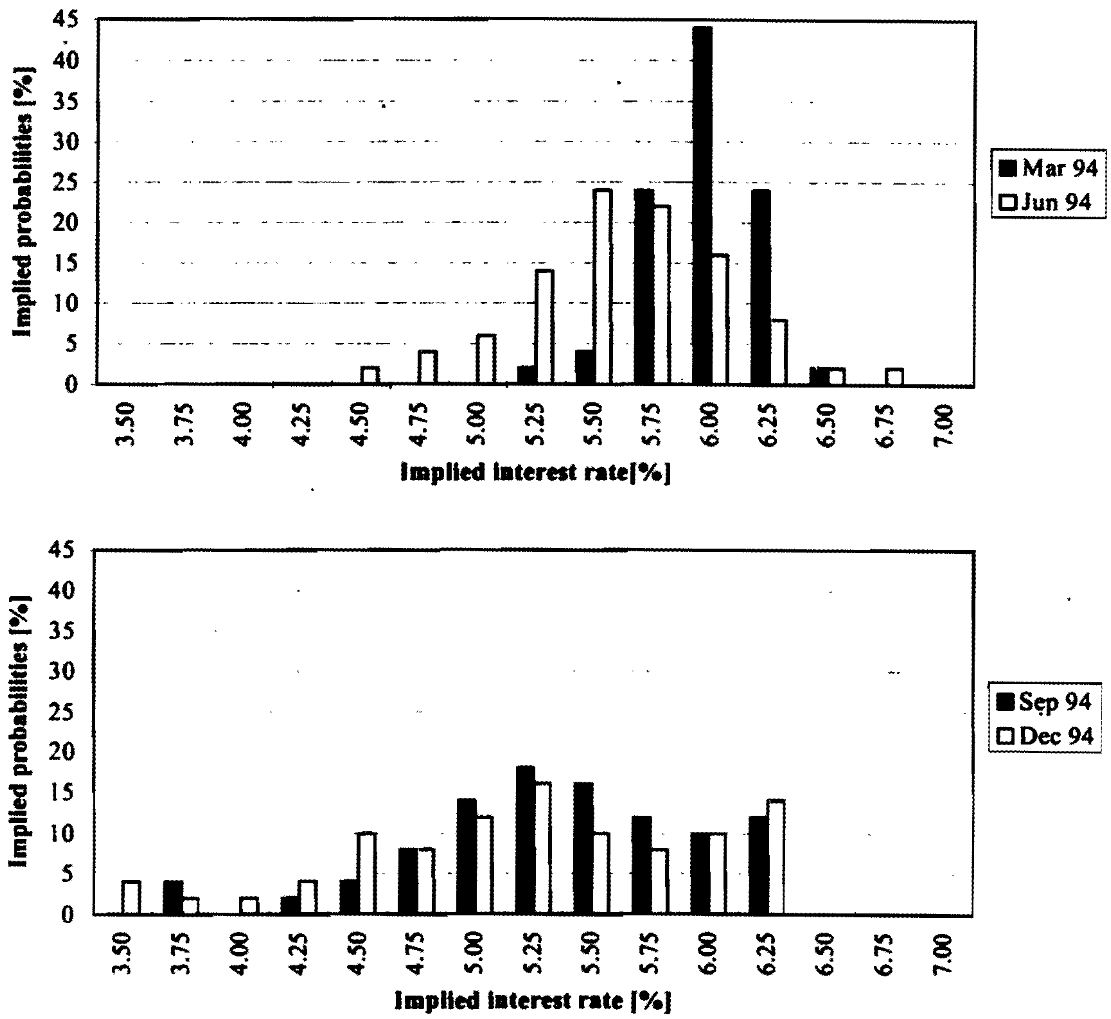
* On the basis of the Euromark future and options on the June 1994 contract. Source: LIFFE, own calculations.
"CBC" indicates that on these days a Central Bank Council meeting took place.

3.2.3 Probability distributions at different future points in time

So far, this study has confined itself to a time series analysis: the implied probability distribution of no more than one contract has been considered over time. The remaining options of other maturity classes have been neglected. Yet, the prices of these derivatives, too, contain information which might be of use to players in the financial market and particularly to central banks.

A possible way of presenting the information is the simultaneous representation of the histograms for several tenors. This is to be illustrated below by reference to the four maturity classes traded on 4/3/94 (March, June, September and December 1994). The histograms, reproduced in two subcharts included in figure 7 for the sake of greater clarity, show two factors in particular: The focal points of the distributions gradually shift to the left, towards lower interest rates, and the ranges of the probability distributions increase with the residual maturity. This last observation is directly understandable intuitively, if one bears in mind that, *ceteris paribus*, more can happen in a longer period. However, the difference in the ranges can only be reflected to a limited extent, since the possibilities of observation are restricted. Thus the small number of existing strike prices for the options with longer residual maturities implies that only probabilities for interest rates of 3.75 % (December contract) or 4 % (September contract) up to 6 % can be determined directly. The probabilities shown in the classes at the edges also contain the probability mass lacking at the edge concerned, and potentially exceed the "true" value of the area actually shown. This is probably the reason why the probabilities in the edge classes rise again for the September and December contracts.

Figure 7: Histograms: implied probabilities of different option contracts; data fom 04/03/94

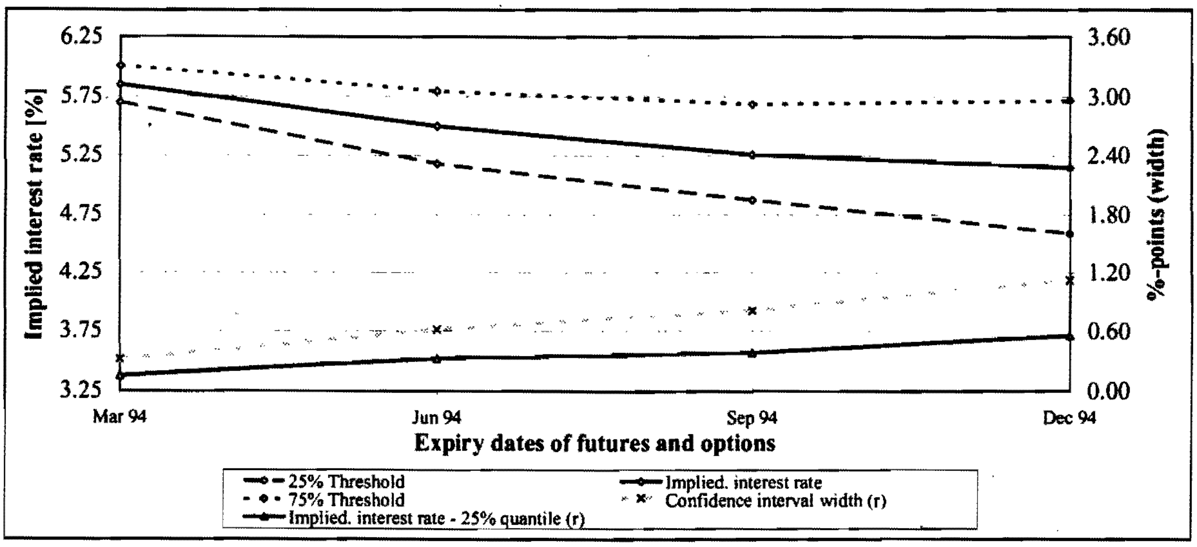


- Indications of class ceilings in each case -

From the observation that the probabilities at the edges are in some cases higher than for the classes closer to the centre of the histogram, it should not be inferred automatically that there are multi-modal distributions. The situation is different with regard to the December contract, the distribution of which shows two (quasi) local maximum values in the classes between 4.25 % - 4.50 % and 5.75 % - 6.00 %. Owing to the discrete approximation, the statement as to whether there is a bi-modal or multi-modal distribution has to be treated with due caution. However, in this case, a normal distribution would apparently fail to reflect market expectations accurately.

Another possibility of summary representation, which, however, does not permit the detection of multi-modality or other irregularities, is based on the variables which were also used in the representation of trends (see figure 8):

Figure 8: Euromark future: implied interest rates and confidence intervals of differing maturity classes[72]



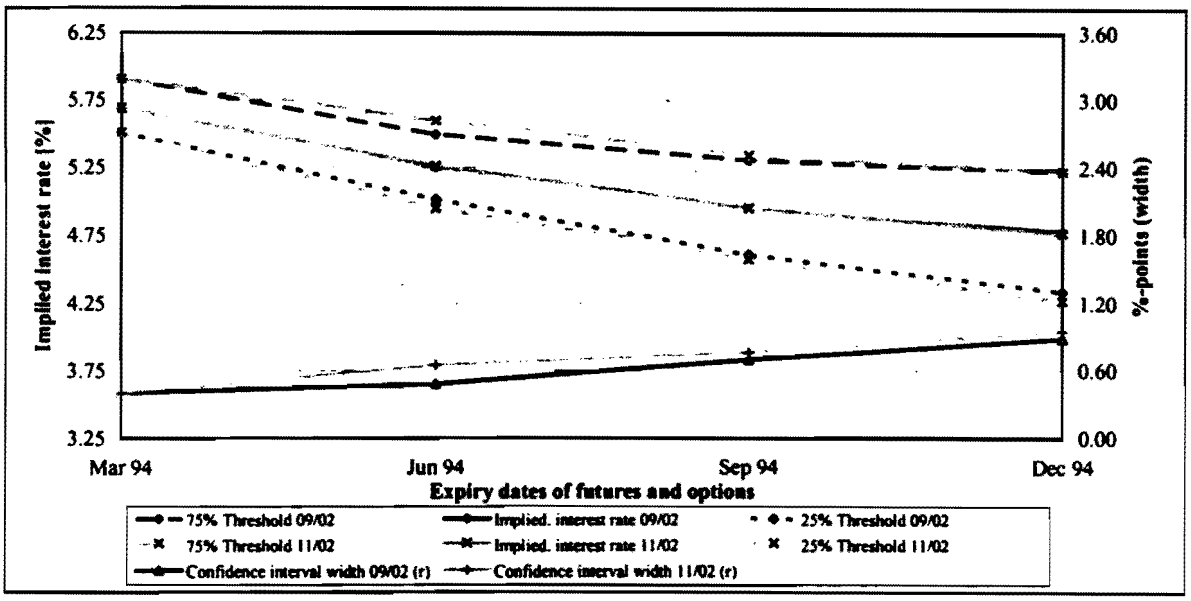
Once again, the interest rate implicitly resulting from the corresponding futures contracts is shown, with the quartiles of the implied probability distributions, plus the consequent interquartile range, which can be subdivided into an upper and a lower part. In this case, too, the increase in uncertainty with the residual maturity is clearly apparent.

72 For variables marked by "(r)", the right scale is relevant.

3.2.4 Probability distributions at different future points of time on different days

The form of presentation chosen in figure 8 also permits a comparison of probability distributions or their position and dispersion parameters on different days. By way of illustration, this is to be shown for a number of days in the period between February 1 and March 31, 1994, which has already been considered. Preferably, a base day should be chosen. If this indicator is to be used for the Deutsche Bundesbank, that could be the date of the most recent Central Bank Council meeting, but also any other relatively quiet or particularly turbulent day. In the present example, February 9 was chosen as a reference date, i.e. the last day before speculation on possible monetary policy decisions pushed the interquartile range upwards in February. The reference distributions on that day are to be compared with those on February 11, when the interquartile range for the June contract reached its maximum value in the period under review. Further comparisons will be made with February 17, when the discount rate was reduced by one-half of a percentage point, and with March 4, which has already been considered. On that day, the implied expected interest rate reached its highest level in the two-month period under review.

Figure 9: Euromark future: implied interest rates and confidence intervals of differing maturity classes and days: 09/02/94 and 11/02/94[73]



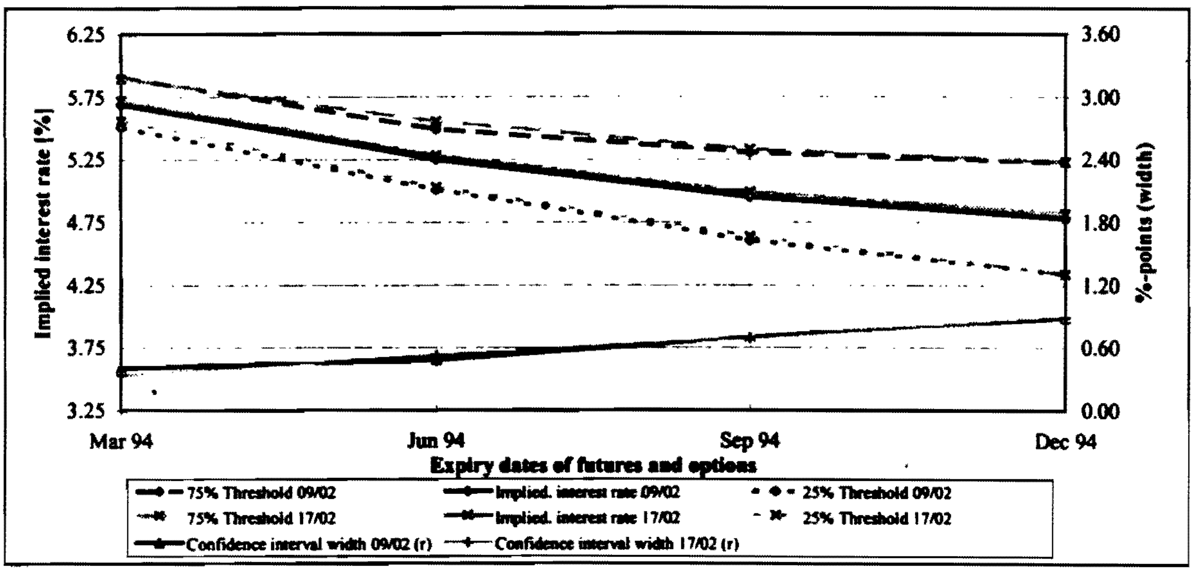
[73] In this chart, just as in the following charts, the reference values are marked in black, the others in grey. The implied interest rates are marked on the continuous line, the 75 % quantiles are on the long-dashed broken line, and the 25 % threshold is is on the short-dashed broken line.

Figure 9 shows the following: on February 9 and 11, the futures values - and thus the implied interest rates as well - were almost identical, and so only one of the two lines is recognisable. A pure analysis of the futures prices only reveals that on both days the market assumed declining short-term interest rates and that the expected values did not change within the two-day period. By contrast, even the simplified representation of implied probability distributions by means of the interquartile range reveals additional information. For instance, the corridor between the thresholds widened over time, particularly for the June contract, and the uncertainty with regard to the September and December contracts increased. The asymmetrical shift in the quartiles is also striking: for the two medium-term maturities, the corridor widened more in the upward direction than in the downward direction. Thus, the implied interest rate in June, which, in accordance with the market's risk-neutral expectations, will be undershot with a probability of 75 %, rose from 5.49 % to 5.60 % within the two days, whereas the 25 % quantile decreased by only 6 basis points to 4.95 %. This could be interpreted as a sign of a change in expectations towards a slower pace of easing of monetary policy.

The situation is different in the case of market expectations regarding the longest tenor. Whereas the futures price and the upper quartile changed only slightly, the 25 % threshold decreased by 7 basis points. Although average market expectations remained virtually unchanged, the visible "extension" of implied probabilities in the bottom area suggests that downward interest-rate movements were now considered to be more likely than upward movements by the same amount.

If one compares February 9 with February 17, 1994, the day the discount rate was lowered (figure 10), one sees that the implied interest rate which derives from the corresponding futures prices rose slightly in all maturity segments. The width of the confidence interval hints at a decline in uncertainty to the level of February 9. The interquartile range for the March futures actually went down even further. This is probably attributable to two factors. One is the Bundesbank's decision to reduce interest rates, which relieved the market of its uncertainty about short-term developments. It was not to be expected that the interest rate would be changed again at the forthcoming Central Bank Council meeting.

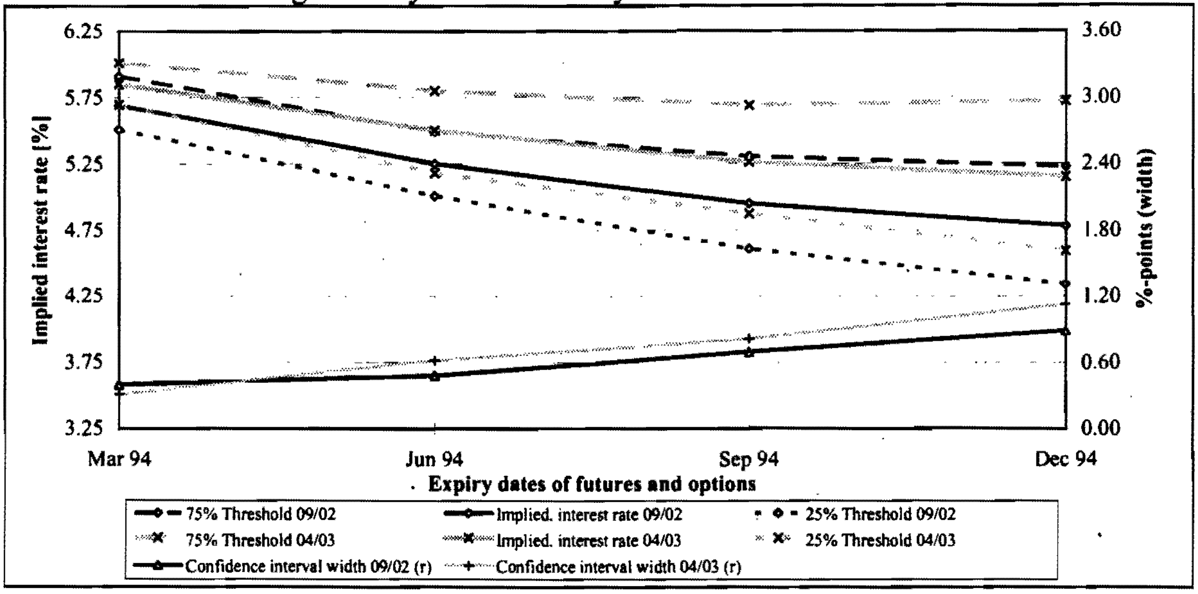
Figure 10: Euromark future: implied interest rates and confidence intervals of differing maturity classes and days: 09/02/94 and 17/02/94



The second factor is the reduced time which remains until the expiry of the futures. A precise assessment of the extent to which the reduction of the maturity is solely responsible for the narrowing of the corridor will be possible only after comprehensive analyses of the data.

A comparison of the March quartiles of February 9 with those of March 4, 1994 (figure 11) also shows a decline in the difference between the 75 % quantile and the 25 % threshold. By contrast, in the three other maturity segments the difference expanded, even though the residual maturities decreased there as well; this suggests that this effect makes itself felt disproportionately strongly in the case of short-term contracts. The main reason for the manifest high degree of uncertainty in the money market on March 4 was presumably the money stock trend that was published two days before, since in the first few days after the publication there was no consensus in the market concerning the future monetary policy stance.

Figure 11: Euromark future: implied interest rates and confidence intervals of differing maturity classes and days: 09/02/94 and 04/03/94



With these and similar methods of presentation, market expectations regarding interest rates can be shown, not only for one day but for a whole sequence of dates. Particularly in cases where changes in such a sequence of implied probability distributions are to be observed, however, the complexity of the data increases rapidly, and makes interpretation more difficult. The specific processing of the data can of course be tailored to the preferences of the users. Thus, alternatively to the forms of presentation used in this chapter, several trend charts (one for each contract) could be drawn up simultaneously, such as were presented in sections 3.1.2 and 3.2.2.

Although an exact representation of the probabilities was (largely) dispensed with, it is possible, by means of a simplified presentation of the quartiles and the differences between them, to make statements on the probabilities of a variety of events. As a result, these variables are very much more informative than, for example, point estimators, such as simple forward and futures prices.

V. Conclusions, and potential applications of the indicators

In the present paper it has been shown that the prices of European-style options can be computed either directly by assuming a probability distribution for the price of the underlying asset on the expiry date, or indirectly by means of the assumption of a random process. In the third chapter, this perception was used to recover the expected parameters of the assumed random process from given option premiums. In the fourth chapter, a new distribution-free method of determining implied probabilities was presented.

The estimated process parameter derived from option prices - the implied volatility - was subjected to an in-depth empirical analysis on the basis of LIFFE data on Bund futures options. The relationship between historical volatilities, various implied volatility measures and - after measuring these - actually realised volatilities (HV, IV, FV), 20, 40 and 60 trading days before the expiry of the options, was analysed. As expected, the impact of historical on implied volatilities is significant, but they cannot be regarded as the sole determinant. This is consistent with theoretical considerations suggesting that option prices include market expectations regarding the future price volatility of the underlying asset.

The empirical analysis, however, went beyond the description of market expectations and examined whether implied volatilities are also suitable for forecasting future volatilities. Forecasts, based on implied volatilities, of the direction in which the dispersion price fluctuations of the underlying asset would move turned out to be reliable. However, the results of quantitative forecasts were disappointing. Even the highest degree of forecasting reliability obtained for a residual maturity of 40 trading days, measured by the coefficient of determination, was a mere 44 % or less. When the residual maturity was reduced, historical volatility, which by definition relates to the past, actually proved to be a more reliable indicator. If, in the regression analysis, historical volatilities were included in addition to their implied counterparts, the explanatory power increased with every reduction of the maturity; but the adjusted R^2 did not exceed 45 % for any IV measure. Moreover, the regression series confirmed the superiority of historical volatilities if a short forecast horizon was chosen. Even though the forecast errors sometimes assumed considerable proportions, the t test for the simple quantitative forecast did not exhibit a systematic error for any of the implied volatility measures. This suggests that the actual option premiums do not deviate systematically from their fair value. In other words: there seem to be no risk premiums.

The findings of the study as to whether players in the option markets may perhaps concentrate on the immediate future are ambivalent. Some implied volatilities measured 60 trading days before the expiry of the options explain variations in the volatilities actually realised in the subsequent five trading days (FV5) to the extent of over 60 %. On the other hand, the coefficient of determination for all other maturities was considerably below the value reached when using FV as a dependent variable, which in contrast to the aforementioned result suggests that market players tend to be geared to the "long term".

In short, it may be stated that the implied volatilities of Bund futures options are useful for describing market expectations regarding the price or rate volatitily of the underlying asset. In addition, they contain information on the subsequent actual volatility trend. This is reliable, in particular, if the only question asked concerns the direction in which volatility in the Bund futures market is likely to move. For these and other forecasting purposes one of the three IV measures CALL, KAPPA or MEAN should be used: they exhibit the best properties. If data processing efforts are to be kept as low as possible, the first variable would seem most appropriate, i.e. the implied volatility of call options that are at-the-money or just out-of-the-money.

A possible reason for the low degree of reliability of quantitative forecasts may be, above all, the continuous inflow of news, which necessitates ongoing price or rate adjustments and which greatly complicates the generation of reliable forecasts. Another reason may be that market players, while using the Black-Scholes model as a common language, as it were, for communication purposes,[74] are not convinced of the reliability of the model and therefore adjust prices manually or calculate them by other methods. To avoid these difficulties, a more general option pricing model was used which does without the assumption of particular random processes and which is based directly on the probability *distribution* of the price of the underlying asset. Thereafter, a new method was presented, with the help of which the probabilities implied in a series of option prices can be determined by approximating the first partial derivative of the option price with respect to the strike price. In this way, the probabilities expected by market players of the price or rate of the underlying asset on the expiry date being within, above or below specific intervals can be calculated. The distribution-free approaches used so far for determining implied probabilities, which are based on methods of the Breeden-Litzenberger type (and thus on an approximation to the probability *density*), are frequently faced with the problem that they can assign to the price or rate interval for which data are available a probability sum of less than 100 %. This makes it necessary to make assumptions as to how much of the missing probability is to be assigned to which edge of the discernible interval. On the other hand, such assumptions are obsolete in the approach presented here, as it is based on the approximation to the probability distribution. The calculation of this function directly

[74] See MALZ (1994).

brings out how much of the probability mass is beyond the upper (or lower) edge which is determined by the highest (lowest) strike price of the options' maturity class.

Technically speaking, the advantages of the indicator "implied probabilities" comprise, besides its forecast horizon (which, depending on the market, is up to one year), its daily availability, a fully adequate degree of up-to-dateness for most monetary policy purposes and international comparability. Thus, on LIFFE alone, besides the instruments for the German bond and money market, options are traded on British and Italian government bond futures and on futures for dollar, lira, Swiss franc and sterling three-month funds.

In line with theoretical considerations, an analysis of implied probabilities derived from options on futures obviates in principle the need to look at the underlying asset itself, since the expected value resulting from the implied probabilities is identical to the futures price or rate. The future is thus redundant. Owing to the edge class problems described in the fourth chapter, which are primarily relevant to Bund futures options, and on account of the distribution within the observable classes (which is not precisely known), the present paper has made use of the futures price as a complement. In the case of options on Euromark futures, this form of presentation was retained on grounds of consistency, even though no data problem was involved. The small difference between theoretically and empirically ascertained values is a further sign of the reliability of the indicator "implied probabilities".

Using the ascertained risk-neutral probabilities, exact statements can be made on market expectations. With the aid of implied probabilities it is possible not only to determine the future values expected by market players. The approach goes beyond this, and makes it possible - depending on the data situation - to calculate quantiles or uncertainty and dispersion measures, such as the interquartile range, standard deviation or expected range. Even moments of higher order, such as the kurtosis of a distribution, can be calculated.[75] It is also possible to perceive whether the implied probabilities are distributed multi-modally, which can prevent misassessments of market expectations, such as may occur when using point estimators.

The knowledge derived from the implied probabilities may be of major importance in preparing monetary policy measures and deciding the best timing of such action. For

[75] However, the interpretation of these data involves difficulties since, in this context the random *process* which determines the *distribution* function may be of significance. If the price of the underlying asset has jumped (jump process), the higher moments are not very instructive.

instance, a central bank may intervene as it sees fit to remedy "undesirable" uncertainty in the market. Given the flexibility of the method devised, the nature of the market - whether money, bond or foreign exchange market - is of secondary importance. Such intervention is not bound to be synonymous with interest rate decisions or interventions. In many cases pertinent press releases or statements may perform the same purpose.

Moreover, implied probabilities enable the success of monetary policy measures to be monitored. Thus, implied distributions provide a visible record of whether monetary policy makers have succeeded in stabilising expectations. Conversely, they also indicate whether the expectations of market players have changed as a result of the announcement of an interest rate measure, either because they have been taken by surprise or because they have now gained greater clarity about the prevailing monetary policy stance.

Another possible field of application is the support of money market management operations, in particular when it is a matter of deciding whether to use a variable-rate or a fixed-rate tender. If the probability distribution suggests massive expectations of an interest rate reduction, a corresponding attitude on the part of bidders has to be expected. If the Bundesbank is not prepared to allow the repo rate to drop perceptibly, a fixed-rate tender should be conducted.

In addition, the use of implied probabilities seems to make sense in risk management or for banking supervision purposes. With their help, it is possible to check whether the potential for price changes derived from historical figures by risk management divisions and assumed in in-house models is consistent with the prices expected *ex ante* by the market. If the potential price changes expected on the basis of historical figures are too low, this indicates a need to adjust.

All in all, implied probabilities constitute a financial market indicator which can be used flexibly and the presentation of which is adjustable to different purposes. Although only a few of these presentation possibilities have been shown in this paper, the empirical analysis carried out as part of this study indicated clearly that even the roughly simplifying presentation of confidence intervals and their width reflects the changed expectations and uncertainty in the markets without having to rely on explicit statements by financial market players. As a matter of fact, that indicator is much more reliable, since it is geared to a variable which is among the most dependable in liquid financial markets: it is based on price data, and thus on the "equilibrium opinion". Therefore, option price data should reveal the market players' opinion in an objectively comprehensible way and more reliably than an inquiry addressed to market participants. One logical corollary of this is that central banks, if they wish to measure market expectations in an undistorted manner, should not engage as players in the derivatives markets in order not to mar their information sources.

In spite of the length and comprehensiveness of this study, there continues to be a substantial need for further research. After all, this analysis has ignored entire derivative classes; but even if options alone are considered, there are still gaps in the knowledge currently available. For instance, in the field of implied volatilities the question arises: how suitable these are as predictors of price fluctuations in Euromark futures or other futures on short-term interest-rate contracts. It would likewise make sense to examine whether implied volatilities, and thus also option prices, are systematically too high or too low over time. This would help to identify the existence of risk premiums.

In the field of implied probabilities, too, some work remains to be done. For example, it seems to be appropriate to use the available historical option data to subject the behaviour of implied probabilities and the variables derived from them, such as the confidence intervals and their width to an even more detailed examination. This could also include an analysis of the forecasting performance of that indicator, or its use in other markets. In addition, it is possible to compare the uncertainty measure "interquartile range" used in this study with other dispersion parameters.

Moreover, it might be informative to compare various methods of determining implied probabilities with the aid of identical data records. Since the technique presented here is able to ascertain implied probabilities without assuming a concrete distribution function, it might provide a reference method of determining the expectations of market players.

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Annex

Selected formulas

1) FED60: **(A1)** $FED60 = \sum_{L=0}^{4} \omega_L \cdot CALL_L$ with $\omega_L = \frac{\left(\frac{Y}{60}\right)^L}{\sum_{i=0}^{4}\left(\frac{Y}{60}\right)^i}$.

ω_L: Weight of the implied volatility (CALL) L days ago

L: Lag (in days)

Y: Residual maturity of the option (60, 40 and 20 days)

2) FED80: **(A2)** $FED80 = \sum_{L=0}^{4} \omega_L \cdot CALL_L$ with $\omega_L = \frac{\left(\frac{Y}{80}\right)^L}{\sum_{i=0}^{4}\left(\frac{Y}{80}\right)^i}$.

ω_L: Weight of the implied volatility (CALL) L days ago

L: Lag (in days)

Y: Residual maturity of the option (60, 40 and 20 days)

The following papers have so far been published:

May 1995		The Circulation of Deutsche Mark Abroad	Franz Seitz
June 1995		Methodology and technique for determining structural budget deficits	Gerhard Ziebarth
July 1995		The information content of derivatives for monetary policy – Implied volatilities and probabilities	Holger Neuhaus
February 1996		Sectoral disaggregation of German M3*	Vicky Read

* Available in English only.